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CONSIDERATIONS ON THE THEORY OF DEMAND – MAIN PREMISES AND CHARACTERISTICS OF STATIC CURVES DIN DSO

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1. An analytical study of the relations between price and quantity demanded which will lead us back to understanding the meaning of the process of individual choices requires some preliminary investigation into the factors and characteristics of such choices. The utility of goods or analogous and more complex concepts (indices of ophelimity) which one resorts to in

This paper was originally published in Italian as «Considerazioni intorno alla teoria della domanda, Parte II – Le principali premesse e caratteristiche delle curve statiche», in Giornale degli Economisti e Rivista di Statistica, vol. LXXIII, year XLVIII, 1933, pp. 765-807; translation edited by Marji Lines.

It was preceeded by another article, which contains a series of introductory critical remarks. This earlier article appeared as: «Considerazioni intorno alla teoria della domanda, Parte I – Rilievi critici sulla determinazione del prezzo», in Giornale degli Economisti e Rivista di Statistica, vol. LXXIII, year XLVIII, 1933, pp. 30-48. theoretical constructs, represent the basis of different concepts which are in part contradictory. Therefore, there are subtle problems as to the relation between the theoretical tools employed and concrete reality. Moreover, there is the not new but always serious problem of clarifying why and in what instances one may legitimately assume the hypothesis of a single market price: a hypothesis that allows one to derive from the single demand curves one that is valid for the entire market.

Before discussing the consumer's economic equilibrium it might then be in order to briefly examine the above-mentioned questions and identify the points of view which appear to be more correct. To this effect we shall follow this line of reasoning:

a) Firstly, we shall review (sections 2-5) the criticism against employing «marginalistic» methods in the analysis of individual behaviour; we shall also mention the doctrines which apply the concept of utility to social phenomena;

b) we shall verify the basis of utility itself (sections 6-7); and we shall then be able to specify the static nature of the demand curve (section 8):

c) finally, we shall discuss some opinions regarding prices which vary for successive quantities exchanged of a given good returning to the problem of the determinateness of contracts (sections 9-10).

Thereafter, the ground will be clear for the study of the elasticity of demand, which we shall try to conduct according to a simplified but at the same time general and comprehensive procedure capable of accounting for certain exceptional cases.

2. In order to consider first of all the arguments put forth by critics of marginal utility, it is advisable to start from a paper by J.A. Hobson who efficiently develops them in order to show that - after Jevons - economists did not succeed in completing the master's system, nor in resolving the main problem of economics i.e., that of collective welfare'. Hobson argues that since Jevons placed the concept of utility, or satisfaction at the theoretical centre, and since «marginal» utilities were valued according to the quantities of goods available, a body of doctrines should have been developed to compare - more satisfactorily than Jevons had done - individual satisfaction with the related psychological costs. This would have served, furthermore, to «humanise» theory by solving the problems of distribution (through qualitative evaluations) and to a picture not only of the economic welfare but of the «total welfare»². This development has not occurred. On the contrary the

¹ A. Hobson, Neo-classical Economics in Britain, Political Science Quarterly, 1925, pp. 337-383.

² On the necessity to unify the concept of welfare, see Hobson, Wealth and Life, London, 1929, chapter III; L. Robbins, Nature and Significance of Economic Science, London, 1932, p. 11, footnote. According to Pigou, The Economics of Welfare, 3rd edition, p. 20, inmarginalistic method was perfected in its exterior form (Wicksteed, J.B. Clark, Davenport, Pigou, etc.), with the disguised intention of justifying, by the mere fact of their coexistence, some typical phenomena of present day society and in particular rents and quasi-rents', Hobson feels called upon to condemn the method itself as inconclusive or as an untrustworthy weapon of conservatism.

Hobson buttresses his argument by adding specific arguments against the concepts of productivity and marginal utility. As to productivity he basically restates what he had already said in his previous works and had been, at least in part, refuted by Marshall⁴: that there is no direct relation between marginal productivity and the remuneration of single factors of production; and that the market supply price does not coincide with the cost of the marginal product but is equal, instead, to the normal, or average, cost of the representative firm which all existing firms aim at as the ideal production unit. This latter proposition could be reconciled with Marshall's doctrine were it not linked to the statement that, in static curves, what matters is only the knowledge of such ideal conditions of production. This excludes all marginalistic criteria, since marginal adaptations could not be theoretically taken into consideration except for a (dynamic) alteration in the entire process of productive organisation.

Analogously, Hobson states – with regard to utility – that the process by which the marginal utilities of the various goods are compared and set proportional to prices is specious and unrealistic; and that attention should be placed instead on the standard of living, seen as a «unit», which represents the result of an «organic» evolution for each individual – an evolution that is basically the same for all individuals belonging to the same social class. He also thinks that it should be ruled out that marginal modifications, as postulated by mathematical theory, occur in infintesimal degrees. In synthesis, this is the thought of one of the most perceptive and lucid critics of the theories under examination. It includes and effectively connects all the objections that have been raised in this regard: those concerning the application of infinitesimal calculus, which is seen by some as an inadequate tool for the study of economic phenomena; those which deny the validity of comparisons

stead, there exists a parallel between economic welfare, which therefore depends on economic factors and on overall welfare.

³ Hobson, *Neo-classical*, cit., p. 355, marginal calculus has attempted to render economics «a serviceable instrument for the defense of the existing economic system by displaying the economy and harmony of its normal working».

⁴ Marshall, *Principles of Economics*, p. 409 and following footnote; cfr. Edgeworth in *Pa-Pers Relating to Political Economy*, London, 1925, pp. 381-384. The use of the concept of «marginal productivity» is necessary for the study of the firm's equilibrium; by analogy what will be detailed later about the nature of the second derivative of the ophelimity function is also relevant. Contraction of the way of the

between the various marginal uses of income (or monetary capital invested in between the various marginal deal the unitary essence the economics of the production) and having verified the unitary essence which each the consumer (or the firm), find revelant only theories which seek to understand the historical development of such an economy'; and finally those which susthe historical development of the behaviour of the individual is both sterile and useless, and lament that such a theory has nothing to say about welfare comparisons between individuals.

3. Some of these criticisms can be easily refuted. Thus, the fact that individuals have no real cognizance of or are unable to perceive the utility of all amounts and all combinations of goods⁶, theoretically does not invalidate the use of infinitesimal calculus. Pareto has underlined this point vividly and repeatedly rejecting those objections which relate more to the singular configuration of specific instances than to the true and inner essence of the theory. In fact, the latter considers the case of extremely divisible goods because it often occurs that divisibility is particularly accentuated and because it is sometimes possible, with obvious adaptations, to explain the exceptions within the same theoretical hypothesis. Even leaving aside the fact that the above-mentioned difficulty disappears when the total effects inherent to the market rather than the individual consumers are taken into consideration. the very same individual buyer can still resort - at least for some specific purposes - either to the valid instrument of calculus or to a clear estimate of the curves of indifference. In general, there are in fact different qualities of goods which cannot be divided, being such that between one and another there is only a slight difference in price and therefore it is possible to allot variable quantities of money (variable either continuously or almost continuously) to the purchase of a given «type» of good7.

⁵ As is well known, besides some isolated authors (O. Spann) it is the American institutionalists (Veblen, Mitchell, etc.) who have stressed this tendency. They do not limit themselves to a critical revision of the traditional theories (as is the case for instance with Knight) but they deny any validity of pure theory. In this regard, see the study by J. Viner, The Utility Concept in Value Theory and its Critics, Journal of Political Economy, August and December, 1925; an unfounded criticism is that of B.M. Anderson (refuted by Viner, p. 384) according to whom, by resorting to the price it should instead explain, the theory of marginal utility appears flawed by a petitio principii. For an historical evolution of the theories on utility see also the lessons given by G. Pirou, L'utilité marginale, Paris, 1932.

⁶ Still to this day many authors grapple with this difficulty: thus the members of the *new* Viennese School, about whom see the recent and learned article by P.N. Rosenstein-Rodan, La complementarietà: prima delle tre tappe del progresso della teoria economica pura, Riforma sociale, May-June 1933. But regardless of its deficiencies - especially relating to the whierarchies» of the goods - the hypothesis of continuity makes it possible to embrace the phenomenon in its basic foundations.

⁷ In a system of indifference curves, for the various combinations of money spent for a certain type of good and that invested in other goods (considered in their totality) the point of equi-

Nor is mathematical theory weakened by the observation that both the economics of the consumer and the economics of a firm are to be considered together. This requirement has been taken into account by the mathematical school – at least in its more developed form – by assuming, for instance, that as far as the consumer is concerned, ophelimity is a function of goods available. However, the function is to be understood as relative to the «individual» who is the basic element in the study of equilibrium and its possible variations (see my first article in Giornale degli economisti, Jan. 1933, par. 3). Here lies the crux of the matter: even though influenced by eminently social factors⁸ and in part depending on the prevailing taste of the group to which the subject belongs, the real taste of any economic subject is autonomous to such an extent as to allow - both for the individual and for the entire market - a general framework of forces and movements by which the equilibrium can be explained and determined. It appears therefore that the theoretical study can be effectively conducted: which of course does not mean to deny the importance of statistical studies or historical-sociological observations whose nature and ends are profoundly different?.

Nevertheless, the problem is more vast than it may appear at first sight. Admitting that statics must consider possible departures from the equilibrium as autonomous (for each individual), it is necessary to determine the extent to which economic behaviour is rational. This is a subject to which we will return in the following sectionss. In the meantime let us note that according to the now prevailing opinion of scholars¹⁰ personal tastes do not

librium lies where a 45 degree line passing through the initial point becomes tangential to one of these curves. Such a representation, which obviously presupposes a knowledge of all the prices is useful in explaining J.B. Clark's statement (*Principes d'économique*, Paris, 1911, pp. 99-103) that, if a given good presents different utility (being of a superior quality), the buyers deprive themselves of a sum of money that for the part corresponding to the last added utility – with regard to a similar good which is of an *immediately* inferior quality – is exactly proportionate to the obtained utility. Analogous remarks may be found in G. Borgatta, Di una proprietà generale dell'ofelimità, *Atti della Accademia delle scienze di Torino*, 1911-1912, p. 17 of the off-print. In his article On the demand for Rival (or Substitute) Commodities, *Econometrica*, 1933, pp. 181-189, Ricci illustrates these concepts with a special graph where the (supposedly independent) utilities of the competing goods are compared with the utility of the money needed for their purchase.

⁸ See among others L. Einaudi, Osservazioni critiche intorno alla teoria dell'ammortamento dell'imposta, Atti della R. Accad. delle scienze di Torino, 1918-1919, para. VIII, pp. 1086-1088; R.A. Seligman, Principes d'economie, Paris, 1927, vol. I, pp. 218, 225, etc. Distinguishing the various means of the terms value and utility. Seligman incorrectly calls social marginal utility what is actually individual and corresponds to the quantities exchanged on the market with the participation of all individuals.

⁹ All that concerns the change in tastes over the course of time, sometimes as a result of political intervention, is not a part of pure theory: Robbins, *op. cit.*, ch. V, sects. 4-6, and especially pp. 115 and ff.

¹⁰ U. Gobbi, Trattato di economia, Milan, 1923, vol. I, p. 2; Robbins, op. cit., p. 89. Previously, Edgeworth, Mathematical Psychics, London, 1881 (1932), p. 16.

necessarily have any egotistic motivation; nor are they bound to any particular hypothesis about the quantity of utility, since it is sufficient to refer to desire (Wicksteed) and classify all the possible combinations of goods (Pareto's index-function) according to an order of preference. Thus, some questions as to the nature of utility¹¹ and its relation to the satisfaction of the senses¹² are also eliminated. Moreover, it appears unnecessary to insist on the fact that equilibrium theory in itself reflects the real conditions of production and the distribution of wealth, but does not attempt, nor is it an adequate instrument to justify the capitalist system although Hobson wishes to demonstrate that such is the theory's aim.

4. Thus we arrive at the difficulties concerning the collective welfare. The accusation of insufficiency which, on account of these difficulties, has been levelled against the theory of utility (for the individual) is not in itself significant because Hobson¹³ and the other critics of the above-mentioned theories do not manage to adequately solve this serious problem themselves. In fact, in order to put the problem well into perspective it is necessary to accept the theories that have been criticised. To overcome the «no bridge» obstacle is indeed impossible in the field of pure economics. Or better, the connection between different phenomena, the search for causal and interdependent relationships, the determination of equilibrium – which are the fundamental tasks of pure economics have nothing to do with an evaluation of facts over all individuals: since this evaluation requires a criterion able to compare the sensations of different individuals.

But a complete theory of utility tells us that while there is no universally valid objective criterion for comparisons based on facts or on human nature; there may, nevertheless, be found countless criteria in relation to the concrete ends which «collective»¹⁴ utility realises. It is therefore useless to dis-

¹¹ Some are mentioned by A. Fanfani, Natura e concetto di bisogno in una visione integrale della realtà economica, *Economia*, Sept. 1932.

¹² This should include satisfactions of an intellectual nature or those of desires stemming from emotions (vanity, etc.). At the moment of exchange, the only moment that concerns the ory, satisfaction is anticipated: it may actually occur and in a quite unforeseen manner, at considerable distance in time (see below sect. 5; and Viner, *op. cit.*, pp. 647-650; Pantaleoni, *Erotemi di economia*, Bari, 1925, vol. I, p. 352; vol. II, pp. 28-29).

¹³ See the last pages of the quoted article. Hobson does not appear to interpret Pigou correctly when he asserts (p. 376) that the latter's analysis of economic welfare is limited to the consumer.

¹⁴ As Pareto has demonstrated, any evaluation of utility is to be seen in relation to specific ends. In the case of economic actions, the individual evaluation (of ophelimity) is the one that cannot be questioned; and the conditions which maximise ophelimity for each individual, and within the limits set by the system may be searched for. The same actions however may also be subject to judgments of collective utility (cfr. Trattato di sociologia generale, vol. III, pp. 2111-2135: however, we are interested here in judgments inherent to the maximum utility cuss collective welfare on the basis of arbitrary personal criteria. It is not useless, for instance, to interpret and analyze a certain criterion (for the purpose of comparison) which, in some given circumstances, whether present or past, a ruling class either applies or aims at applying. Nor can it be *a priori* ruled out that such an undoubtedly positive study translates into figures, that is to say in terms of utility, the comparative positions of individuals and that of the entire social structure.

For this reason it appears unadvisable to indiscriminately reject all comparisons of utility of different persons from the field of applied or political economics. If they correspond to conditions and historically demonstrated criteria such comparisons are not unjustified; and it is certain that since they require the knowledge of economic theories and facts, only the economist will be in a position to conduct them. On the other hand it is obvious that because of the very difficulty of verification, it should not be expected that such an investigation is of a rigorous nature.

The old question of proportional versus progressive taxation, for example, cannot be solved without using some of the above-mentioned criteria. These may indeed be called political¹⁵ but whatever they are called it would

for the collectivity, and not of the collectivity as a whole. The way to approximate the maximum collective utility is obviously that of altering the *constraints* which in part depend on the laws and the State's political action.

¹⁵ See the recent article by L. Einaudi, I sommi principi utilitaristici e l'imposta, Riforma sociale, 1933, pp. 474-484. Clarifying what is, strictly speaking (due to taxation), the object of study of public finance, Einaudi includes among the ends of public finance the criterion of distribution of the total burden of taxation (pp. 482 and ff.). The proposition is important since it shows the connection between collection and use of taxes, which are both instruments of national economic policy and result in uneven advantages for the individual classes of taxpayers. In this regard we may inquire as to the impact of the criterion of rationality in taxation recalled by the same author in a previous article (Il cosiddetto principio dell'imposta produttivistica, Riforma sociale, 1933, pp. 378 and ff.). According to Einaudi that distribution of taxation among taxpayers which results in the maximum degree of wealth may be called rational: wealth in relation both to the consumption of income for the attainment of material and immaterial goods, and that of their production (in part depending on or conditioned by public services). Now, in such a scheme, the comparison of marginal utilities - which really is one but not the only aspect of the evaluation carried out by the State since the presumed productivities of the public service must also be considered - is carried out by the State as representative of the individuals: and this representation does not consist in the mere interpretation of the individual scales of preference (something which while not absurd presents serious difficulties and clashes with the no bridge principle), but is carried out through a process of «correction» of those scales which are thus co-ordinated with an organic system of ends. It must be noted moreover that even when the (modified) utility scales are supposed to be known, the determination of a given taxation burden for each individual does not stem directly from them. On the contrary the equality of marginal utilities for the part of individual income devoted, on the one hand to public services, and on the other to private goods cannot be determined unless we know the general criterion used for the distribution of taxation (for no doubt be opportune and advisable to specify at least the most important ones, to reduce them to formulae and then apply them rationally as valid complements to the given economic policy. As is well known, a few steps along this path have been taken both with the various principles of sacrifice16 which although too rigid have an historical political tradition, and with the construction of a marginal utility curve for income for all taxpayers. It has been observed¹⁷ that not only the criterion of sacrifice which one chooses to adopt but also the construction of the utility curve reflect a particular way of comparing the sensations of individuals: in fact, the curve is at least subordinate to the principle of equality among members of the collectivity having certain income¹⁸. But this itself does not appear to determine a weakness in the attempts made so far. The principle may be reasonably adopted in relation to the consumption of income. The attempts made to include the problem of taxation within a «utilitarian» vision, instead, seem so far to have been flawed especially because they do not take into consideration the (psychic) cost incurred in obtaining the income: a cost which varies according to the specific kinds of income (salary, rent, etc.) and must therefore somehow counterweigh the utility, that is, the satisfaction derived from its use.

5. The problem of collective welfare has also been examined by orthodox economists, that is those who do not stray far from the main body of science. Some may indeed exaggerate in calling their constructions unfounded. But actually they do not lack contact with reality and reference to the

various quantities of total revenue). Similarly in the purchase of goods on the market the consumer must know the prices in order to compare the various marginal utilities. Between the two instances however there is the difference that the market prices – which are the same for everyone – are ultimately the result of the competition between buyers, whereas there is no such interrelation in the case of income tax. Thus we can say that this marginal comparison, far from determining how the distribution of taxation takes place, only serves to ascertain, as a kind of test, the coherence and thoroughness of the government's financial plan. And at any rate the test is only a theoretical one since, given the factors involved, it seems unlikely for to be put into practise. Therefore we must limit ourselves to simpler, albeit imperfect, schemes such as those that take elementary criteria that are part of the ends of the State's activity, and deduce the nature of taxation whether it is more or less progressive.

¹⁶ The most comprehensive analysis of these principles is that of R. Frisch, New Methods of Measuring Marginal Utility, Tübingen, 1932, ch. 11.

¹⁷ Cfr. Robbins, op. cit., footnote p. 126 and L. Einaudi, I sommi principii utilitaristici l'imposta, cit., pp. 481 and ff.

¹⁸ As to the equality between owners of *different* income – and equality which appears from the *single* utility curve devised for everyone – I have already had the occasion to discuss it briefly in reviewing an article by Ricci (in *Giornale degli economisti*, July 1932): I envisaged there that the equality would derive from the fact that since it is impossible to measure the utilities of «all» the single portions of income, its utility curve cannot have any other significance except that (through an intuitive process) it includes and links together the hedonistic situations of individuals enjoying gradually decreasing incomes. opinions that appear to prevail in a modern society about the way to compare the satisfaction of the rich with that of the poor. But certainly it must be recognised that in the constructs many procedures and hypotheses do lack a positive foundation.

For example Edgeworth's hypothesis whereby the amounts of pleasure or of suffering constitute for each individual the «perceptible minimum»¹⁹ appears both arbitrary and excessively abstract. Also equally inadequate is Pantaleoni's analogous statement²⁰ that for maximum collective utility one ought to proceed to the distribution of goods according to the «hedonistic sensitivities» of the individual. Pantaleoni's concept of the hedonism of a species²¹ with which it would be possible to evaluate individual actions in relation to the welfare and progress of the collectivity is also too vague.

Nonetheless there remains the well-known concept of Marshall (resorting to the monetary unit to measure satisfaction) perfected by Pigou, with special regard for those fine tools of investigation which are the «marginal demand price», «the marginal supply price to the industry», and the «marginal supply price to the community»²². This is certainly not exempt from criticism, especially when one tries to attribute to the above concept too wide a field of application to it. Although at present it does not provide a flexible and therefore useful criterion of comparison, Marshall's idea appears, nonetheless, susceptible to further development and correction, precisely in view of its simplicity and concreteness. In my opinion, it is above all necessary to clarify the extent and influence of the most characteristic of economic phenomena for each different social stratum; and to then attribute a variable importance to the monetary unit in relation to individual income. Thus, through reasonable hypotheses - which are naturally approximate but nevertheless adhere to a communis opinio, and to the ideals of a liberal, corporative society, etc. - it should be possible to fill the gap between the objective signs of welfare and its measure as a sum of satisfactions²³.

¹⁹ Mathematical Psychics, cit., pp. 7-8; and p. 124 for the comparison of this criterion to another by Sidgwick.

²⁰ Cenni sul concetto di massimi edonistici individuali e collettivi, *Erotemi*, cit., vol. II, p. 29. Pantaleoni's study is inspired by Edgeworth's concept which it tries to interpret through plainer, though not entirely rigorous, arguments (para. III). The most original part consists in the dual classification of hedonistic maximums according to the end and to the means of their realisation, and in the various examPles of para. V illustrating the meaning of such classification. In later writings, Pantaleoni showed his adherence to Pareto's doctrine on the maximum ophelimity for the collectivity (*Erotemi*, cit., vol. I, pp. 219 and ff.).

²¹ Principii di economia pura, 2nd edition, 1894, pp. 29, 38; Erotemi, cit., vol. II, p. 5.

²² See the mathematical exposition in *The Economics of Welfare*, App. III, para. 15-22.

²³ It would be sufficient to determine the *sense* of the variations which cannot be evinced with certainty from only the objective suggestions (Viner, *op. cit.*, pp. 658 and ff.). See also Edgeworth, *Papers*, vol. II. pp. 321 and ff.

6. Let us now return to the main subject of this study: price phenomena and the behaviour of the consumer.

and the behaviour of the constant in obtaining goods the economic subject We have previously stated that in obtaining goods the economic subject has a wide range of choices. It has also been suggested that the difficulty of modifying the consumption of all goods considered ofelimi, when a disturbing factor intervenes, or even of moving from goods of one kind to analogous goods of an inferior kind, is a secondary and therefore negligible phenomenon in a general theoretical synthesis. Now, however, we must see the individual makes his choices rationally; that is if he always tries to satisfy his tastes to the highest degree and whether these tastes must be taken as known²⁴, and in what measure, for the determination of equilibrium. Depending on the answers to these questions in this or that way we arrive at different static configurations of the economy, at least with regard to consumption. We also implicitly give answers to other serious problems, especially concerning the possiblity that the individual may make *errors*.

Through the observation of reality we find instances of actions that are, to a greater or lesser extent, rational. We cannot therefore derive (from observation) a unique and well-defined criterion enabling us to establish with certainty the fundamental hypotheses of the theory. At most we can abstractly conjecture the invariability of tastes through time, therefore leaving out errors, regrets, «economic remorse». Before that limit is reached we can conjecture other hypotheses, e.g. that the individual may act without full consciousness of utility or at times know imperfectly the objective qualities of the goods, being guided in part by intuition, especially when the economic plan refers to a long period of time (the cases of savings and durable goods). We must nevertheless rule out that economic theory may consider the behaviour of individuals as entirely impulsive or unintentional.

²⁴ To assume the tastes to be known amounts to taking them as «factual data». I will later clarify the meaning of this expression; but I should make it clear that the theory of equilibrium though somehow requiring the knowledge of tastes does not require an understanding of how they were formed; which at any rate does not appear to be subject to rigorous laws. Therefore to imagine the individual as «without experience» and to indicate then how he gradually forms his plan and takes into account the complementarities and hierarchies of goods and their multiple uses, is no doubt useful for a didactic systematisation but does not appear to lead to a valid theory, rather to a complementary illustration of economic theory. Nor can it be said that there are two problems: that of a complete, ex novo determination of the individual economic plan and that of mere marginal adaptation with regard to a previous position. There is only one problem because there is one concrete fact that must be explained. The above-mentioned duality has been envisaged especially by Mayer and Schonfeld. Rosenstein (op. cit., pp. 289 and ff., 298, 306 and ff.) considers it a basic innovation in the theory of utility; but it seems to be in conflict with Pareto's observation (Manuale, IV, 67 and Appendix 16) that in order to define a point of equilibrium and to investigate its characteristics it is enough to know the not very extensive area surrounding it.

In Rosenstein's study quoted above, the different formulations of the theory of choices (Davenport, Fetter, Wicksteed, etc.) are clearly dealt with.

The most opportune hypothesis for a theoretical system is to be decided in relation to the elements involved.

If we aim at determining the general economic equilibrium of both production and consumption and for the whole market, we must assume a considerable stability in the tastes of the consumers. In Pareto's conception the stability of tastes is complete: this is coherent in his *Manuale*, the *Manuel* (Appendix) and *Economie mathématique*. Rosenstein's opinion that in the latter two works a new idea on the character of choices is developed²⁵ is in fact debatable. It seems more correct to think that in these works the only concept that is developed is that, for the determination of equilibrium, the notion of ophelimity is not needed.

The Italian edition of the Manuale clearly stated that there are countless functions which may indicate ophelimity in the sense that the relations between the individual and the goods are explained equally well. In the Manuel and in Economie mathématique²⁶ it is shown that this is so true that if these relations are all posited as real (that is having occurred in the past and nonetheless referrable also to the present because of the hypothesis about the stability of tastes), it is enough to know them to deduce an index-function capable of showing how the individual reaches an equilibrium point. In substance, in his last works of pure economics, Pareto simply wanted to accentuate the independence of the theory from psychological premises: starting from the facts²⁷, he showed how it is possible to construct, or reconstruct the index-function. Therefore, this does not mean that choices precede utility or that economic behaviour is irrational²⁸; it only means that the index-function and the concrete choices are in harmony: the first reflects the second. And the rationality of behaviour is the direct and immediate consequence of the fact that it is a question of repeated actions²⁹.

²⁵ Ibidem, footnote pp. 262 and ff., pp. 273 and ff.

²⁶ Encyclopédie des sciences mathématiques, Paris, 1911, pp. 595-597.

²⁷ Sect. 3 of *Économic mathématique* and sect. 5 of Appendix of the *Manuel* clarify how an index-function of ophelimity for the zone surrounding the point of equilibrium can only be obtained hypothesising that there be changes in *all* prices.

As Pareto notes, it must be said that the problem of the order of consumption (about which see *Manuel*, Appendix sects. 14-19) may be left out of consideration because to every combination of given quantities of goods there corresponds a certain mode of use (order and association of consumption) which is considered optimum and therefore adopted by the economic subject. Because of this consideration, the objection raised to Pareto's theory (Rosenstein, op. cit., pp. 278, footnote, 305, etc.) because it had not taken into account that the modes of employment of the goods are unknown, appears to be unfounded. The unknowns in question are uniquely determined by the determination of the purchased quantities.

28 Rosenstein, op. cit., pp. 273-274.

²⁹ Économie mathématique, cit., p. 604: «Economics only considers average and often repeated phenomena». In the absence of indications to the contrary, this sentence must be interpreted as similar ones found in the text of the Italian and French editions of the Manuale.

7. But Pareto's above-mentioned hypothesis of a *complete* stability of tastes does not appear to be indispensible.

First of all, it should be noted that in a certain way it undermines the attempt to eliminate every arbitrary element from the theoretical scheme. In fact, according to Pareto, to deduce the index-functions of ophelimity from the concrete choices is to make evident the experimental character of the theory; but how can the logically necessary premise of this deduction be justified? Isn't it true that it in turn constitutes an arbitrary element which is not subject to demonstration?

Second, it seems advisable to avoid using the above-mentioned hypothesis since it is not only arbitrary but also quite contradictory. It posits the stability of tastes for a period of time that is «not short»; and during which individuals are also supposed to be able to undergo multiple experiences. Now it is easy to see how the variations in income and costs which make possible such experiences are conditions which are not likely to leave tastes unmodified.

It is therefore unfounded to hold that in the very positing of general equilibrium the tastes of individuals, while being connected to past personal experiences, are not *completely* determined by them. A certain margin may also be left to the *intuition* of utility which today is favoured by powerful means of communication (advertising, etc.) and mainly derives from the observation of the behaviour of others. In these hypotheses, the index-function of ophelimity becomes a primary datum³⁰ which precedes choices – and cannot be in conflict with the ones occurring in the time interval taken into consideration – and is moreover subject to a slow and continuous transformation due to the new experiences. Thus, at all moments these never entirely consolidated individual tastes concur at all moments in determining the equilibrium which would be arrived at had they and the other conditions (obstacles) not changed³¹.

The first section of chapter III states: «We simplify the problem if we assume that the subjective fact adheres perfectly to the objective one; and we can do this because we only take into consideration repeated actions; which also enables us to assume that the nexus between actions is a logical one».

The appendices of both the Manuel and Économie mathématique illustrate this concept, which is also expressed in the articles published in the Giornale degli Economisti, 1900 and 1901, Sul fenomeno economico, and Sul principio economico.

³⁰ The datum is primary, ot course, for theoretical purposes. In practise, the comparison of utilities is only valid for marginal adaptations; but since it does occur, it involves the *possibility* of any other evaluation included in the index-function. In his well known polemic with Pareto, Croce failed to recognise this when he denied the scale of subjective values asserting that the individual «is dealing only with an action he has accepted and excludes all others which are infinite for him and represent only *un-chosen actions* (non-values)». Cfr. B. Croce, *Materialismo storico ed economia marxista*, 3rd edition, Bari, 1918, p. 248.

³¹ These considerations complete what I had occasion to present in part I as a note to

But, on the other hand, we should not forget that if a precise norm cannot be given, the basis of experience – assumed, as already stated in a greater or lesser degree depending on the problems in question – must be quite solid and prevalent for general economic equilibrium, lest the theory be almost completely invalidated. In the examination of particular problems, instead, more frequent and rapid changes in tastes may be assumed.

8. Keeping in mind the preceding remarks which aim at establishing the precise foundation of the evaluation of utility, the formation of the demand curve takes on a clearer meaning.

Each of the curves formulated by the economic subject for each individual consumer good reflects the whole of his tastes: the overall view of the utilities is unified and may be stated to be the essential characteristic of the demand curves. But as, on the one hand, new needs arise and on the other the utility of goods gradually changes, the corresponding curves also gradually arise and change over the successive typical intervals of time. The merit of such a conception of static analysis which does not rule out successive modification while leaving aside the consideration of what happens or could happen (virtual movements) during a given interval of time is immediately evident. As Schumpeter says³², the field of statics appears «much widened by the fact that it is not a condition of stasis». It is also clear that among the basic forces of equilibrium (which may be theoretically assumed, but as a tendential equilibrium for each interval of time), those which appear in the demand curves (see Part I) act more readily and definitively. These curves ultimately rule all economic activity and determine market prices. The numerous and more or less reproducible factors of production which in Marshall's scheme should form, together with the related prices, the constitutive elements of the supply curves, are in fact employed in order to satisfy simul-

sect. 5. Since Pareto's static system – unlike Marshall's – ignores any change that mightintervene in the *data* through time and therefore does not appear to be necessarily connected with the hypothesis of average and repeated phenomena. The hypothesis may be relevant if we take «average» in reference to the fact that a certain interval of time is being considered to which all economic operations, even if normally carried out at different frequencies, refer. (See Pareto, *Économie mathématique*, cit., p. 604): and «repeated» is to be understood in the sense that they take place frequently but not to the same extent and with the same modality (for example, the sale of a product).

This opinion of mine is strengthened by the fact that the general economic equilibrium envisaged by Pareto links certain initial positions with some final ones but does not demonstrate that the initial positions (quantities of goods possessed by single individuals) ought to be reconstituted as they previously were.

³² J. Schumpeter, The Instability of Capitalism, *Economic Journal*, Sept. 1928, p. 373. This author (see the preceding pages) understands the supply curve as a list of the various *present* possibilities: this appears to us, instead not significant with regard to the real course of phenomena.

taneously and in the best possible way *all* demand: so that strictly speaking the individual supply curves do not have an identity of their own³³. To put it in other terms, it may also be said that there is only one informing principle of economic action: that of providing for the different ends according to a decreasing order of importance; and this principle, which also informs the area of production³⁴, is first of all followed by the individuals in forming the demand curves.

Some words of caution are here in order. If the good in question has no close relation of interdependence with other goods (a complement to or substitute for the first) it is correct to consider an even not very short span of the demand curve since it is not unreasonable to imagine that the prices of the other goods remain almost unchanged. But if relations do exist it becomes impossible to assume that when the price of the first good changes, the economic subject remains on the original demand curve: the price of the complementary or substitute good varies at the same time in a measure that depends on the actions of all consumers and producers, and the premises on which the first curve had been based also change. In these instances it would be advisable to limit oneself to slight divergences and consider the variations of (individual) demand as due to the variations in the prices of one, then the other good. At most, given these variations, the variation of the individual demand of one of those goods is a total differential whose value depends on the two partial derivatives. The subject will be mentioned again in section 21.

I must also point out that by assuming the variation of a given price and examining the effects on individual behaviour the causes of the variation are not taken into consideration. What is being examined, in substance, is only the configuration of the tastes of the consumers – not excluding the tastes of those who produce the good in question. On the other hand an investigation of the impact of a variation in the initial quality of a good owned by single individuals on the general equilibrium, which takes into account the price and quantities exchanged of the same or other good would be much more complex³⁵. But such an investigation cannot be fruitful unless the exposition

³³ The same may be said of the demand curves for production factors. The conditions and limits with which Marshall surrounds the demand *derived* curve for a production factor (*Principles*, p. 383 and following footnote), deprive said curve of all practical relevance. might be better to indicate in a general way (Marshall, *op. cit.*, pp. 385 and ff.; Pigou, *op. cit.*, p. 262) the elements that, in the hypothesis of a reduction in quantity, affect to a greater extent the variation in price.

³⁴ Schumpeter, op. cit., p. 366, footnote; Robbins, op. cit., pp. 76, 78-81, where the author recalls Wieser's doctrine on the *specific* and *non-specific* factors of production denying that any case the real cost of production is an *independent* principle of explanation of the value.

³⁵ Bordin tried it in his study La legge della domanda dal punto di vista della statica e della dinamica, cit. of the relations between that variation and *all* the processes of production is not excessively abstract. Indeed the variation should first be observed in the quantities of a factor of production (for example raw materials) which although not usually of direct utility for everybody may normally be employed in various ways and for different purposes.

9. The hypothesis of a constant price for units successively purchased – which is at the base of both the individual and collective demand curve – is not justified except in the case of competition on the part of the buyers. My line of reasoning is here connected to the debated problem of the degree of determinacy of exchanges. According to Edgeworth's well known thesis a final and uniquely determined position in the market is reached only when the various conditions of perfect free competition obtain³⁶. This position in the representation of the indifference curves is marked at the point which the individual buyers or sellers arrive at along straight line paths. For Edgeworth, in any other hypothesis there is a multiplicity of final positions, none of which is necessarily arrived at as a result of iterated contracting. In particular, in the most simple instance of a sole pair of buyer and seller – or also of more pairs provided they are analogous – the possible final positions are geometrically represented by the so-called *contract curve*.

Edgeworth's thesis does not seem to be entirely acceptable. One ought to distinguish the case in which competition is not so perfect only on the side of the sellers from that in which it is imperfect for both buyers and sellers. In the first case we think that the terms of trade are determined even when the sellers are not reduced to one person: that is, not only in the case of a monopoly, but also in that of *polypoly*. Excluding the hypothesis of an agreement among all the monopolists – so unlikely that the hypothesis need not be discussed – there is no reason to believe that they should not arrive at a position which assures them maximum *lasting* (rather than occasional or momentary) advantage. In this reasoning we follow Cournot and other authoritative writers such as Schumpeter and Amoroso³⁷. One may want to abandon the hypothesis which rules out any agreement, and taking into account the cost of a competition among monopolists, find the solution most in keeping

³⁶ Edgeworth, *Mathematical Psychics*, cit., pp. 17 and ff.; Jannaccone, Il «dumping» e la discriminazione dei prezzi, *Riforma sociale*, 1914, p. 235.

³⁷ Schumpeter, op. cit., p. 370 footnote; Amoroso, La curva statica di offerta, cit., pp. 13-19. Naturally it is not logically necessary to assume that monopolists arrive at the quantities of equilibrium through decreasing oscillations. Previously. Jannaccone (Questioni controverse nella teoria del baratto, *Riforma sociale*, 1907, pp. 667 and ff.) had advanced an exception to Edgeworth's thesis of contractual indeterminacy – exception based on the monopolists» limitation of available quantities – which Pareto had practically accepted (*Manuel*, p. 602). Cfr. also V. Porri, *Principii di scienza economica*, Turin, 1932, pp. 348-351. with their comparative strengths, which result from the economic and noneconomic data of the problem (contractual ability, resistence, etc.). The solution will be a form of agreement which may simply consist in a maximisation of the total financial advantage of the monopolists while the dimensions and structures of the respective firms remain unchanged, or given the appropriate circumstances, mergers or concentrations, etc. may also occur. In all cases the demand curve subsists as the condition for equilibrium³⁸, and the price is constant for different units of the same good exchanged.

If on the other hand the lack of competition is bilateral the reference to the supply and demand curves as possible equilibrium for either buyers or sellers would be of no avail. In the most simple instance of a sole couple, one buyer and one seller, what is important to know is the contract curve because it is on it and not on the exchange curves (supply demand) that buyer and seller will reach equilibrium. This is the case even though they might at first make reciprocally incompatible offers. It is clear that in this case too, contractual indeterminancy may be eliminated by taking into account the various strengths of those entering the contracts. These strengths include the complex of abilities, availability of money, capacity to foresee. etc., which concur, together with tastes, in determining the operators» inclination to trade. This has been noticed by many authors³⁹. But as long as the notion of these strengths is of a generic kind, it will not be useful, rather it will be tautological to assume that the strengths of those contracting are given and comparable – in the form of a relation – and thus arrive at a point of equilibrium on the contract curve. It would also be wrong to assume that the individuals are more than two and with different tastes, and then define in some way the paths (sequences of variable prices) which lead to the equilibrium point. Since recently Bordin has sketched a theory to this effect⁴⁰, shall now try to emphasize its weaknesses.

In short, Bordin holds that the relations between the hedonistic strengths of contractors(as defined above) when known by repeated experiences, may allow one to determine on the indifference curves representing all possible pairs of individuals, the direction followed through the exchange of infinites-

³⁸ In the case of monopoly however Pareto does not simply consider a demand curve: he considers it possible that the prices of the other goods do not change, thus establishing a relation between the monopolist's price and the quanity in demand (on the market) provided that the prices and the quantities of all other goods exchanged also changed (*Manuel*, Appendix, sect. 68; cfr. Edgeworth, *Papers*, vol. II, pp. 454 and ff.): that is to say, he adopts the point of view we had suggested at the end of the Preceding paragraph.

³⁹ Among others, Pantaleoni, Analisi del concetto di «forte e debole» in economia, Erotemi, cit., vol. I, pp. 349 and ff.; Jannaccone, Questioni controverse, p. 650; Schumpeter, op. cit., p. 371. See also Edgeworth, Mathematical Psychics, cit., p. 56.

⁴⁰ A. Bordin, Schema di varianti della teoria paretiana dell'equilibrio, Annali di economia, 1930, and Lezioni di economia politica: la statica (Part I), Padova, 1932, pp. 167-186. imal quantities. Then in the area where the contractors derive some advantage from the exchange, a line may be found which indicates the path followed until every advantage to be gained from exchange is exhausted. And this holds for whatever number not only of goods but also of individuals.

No objection can be raised against the application of variable prices in the case of an occasional meeting, occurring every so often between two individuals who are exchanging two goods (one of which may be money). Marshall had already given a clear example of this in his Nota sul baratto even though he did not introduce a mathematical relation between the socalled «hedonistic strengths» of those contracting. Moreover the same scheme may also be used in the case of repeated meetings again between two agents and under unchanged conditions. But since, at least hypothetically in every encounter the same point on the contract curve is reached, it is of negligible significance to know the path followed in the preceding attempts at adjustments to reach that point. And the same may be said in the instance of many equal contracting pairs if the contractual meetings are assumed to have been repeated over a period of time⁴¹. For any other instance of repeated meetings no application of the concept of variable prices may be possible, even assuming that the conflict between the supposedly unchanged «hedonistic strengths» produces and maintains an equilibrium point. The contractors in fact, once the disorder of the first trial has been overcome, all agree to the exchange of given quantities (for each) of the two goods in question, and by the same agreement eliminate any connection with the preceding attempts. It must be noted that from these attempts no uniformity concerning the variable prices may be derived since, even given equal results one can have the most various network of exchanges between individuals of both sides, with the corresponding changes in the variable prices. What may be said is that in both occasional and repeated meetings the contractual transactions will not cease until the ratio of the marginal utilities of the two goods is equal for all operators, but in both instances this may occur in countless ways. In the face of the evidence of these observations it seems superfluous to conduct an analytical study of Bordin's mathematical formulation: the critique of the premises on which it rests may suffice. It is also obvious that the same critique is a fortiori valid for the general case of n number of goods and in the hypothesis hinted at by Bordin of variability in the relations between the hedonistic strengths during the implementation of the exchanges.

Thus we come to the conclusion that, if the number of individuals on both sides is small or if the equilibrium reached by the market – because of

⁴¹ In an occasional meeting, even admitting that the n sellers of a certain good are all equal as are the potential n buyers (eventually offering money), it does not occur that an individual enters into contract with only one other and, therefore, a single contract curve does not exist. See footnote 1 to para. 4 of part I.

their separate actions – is purely accidental rather than stable, given the possibility of changes both in the pairs that engage in the exchanges and in the number of the latter; or if the individuals, having become aware of the respective strengths try to keep the *total* result of the exchanges in line with them⁴², so that the necessarily collective contract has a character of greater stability since it takes place over a period of time that is not short – then one must exclude any consideration of variable prices and merely determine the quantities exchanged by each contractor.

At this point it must be added that the case of a lack of bilateral competition is actually very rare. No doubt in theory an essential condition for perfectly free competition is the practically unlimited divisibility of the two groups, that is there must be a considerable number of them and each must have at his disposal small quantities of goods in relation to the total amount. As Edgeworth demonstrates, iterated contracting makes it possible to arrive at an intersection of the supply and demand curves only when the number of supposedly equal individuals – on both sides – approaches infinity⁴³. To verify this it would be easy to show that once the point of intersection has been reached, any individual who exchanges his good at that point is able to improve his position (that is to say that he can move in a direction rather different from the marginal one through a constraint on the quantity of the good supplied and the subsequent price increase⁴⁴ whenever the ratio between that quantity and the total market supply shows some positive value. It may seem then that competition is almost always lacking on both sides.

⁴² Pantaleoni states that «the contract is founded on a presumed equality of forces» (*Erotemi*, cit., vol. I, p. 359), and if the forces are initially unequal, the advantage of a contract arises at that point in which «the employment of further and greater force for the person who incurs the cost is perhaps an increasing cost no longer covered by the utility of the result» (*ibidem*, pp. 92 and ff.). Cfr. also Jannaconne, *op. cit.*, pp. 659-663 and especially the observation that the ends which at times competitors aim at is that of causing the others to gain as little as possible.

⁴³ Mathematical Psychics, cit., pp. 35-38. In other works by Edgeworth the process of iterated contracts is further clarified; see in *the Giornale degli economisti*, October 1891, the short article Ancora a proposito della teoria del baratto, where emphasis is given to the logical rather than to the descriptive import of reiter ation (p. 317); and likewise in *Papers*, vol. II, pp. 311 and ff.. Here it is stated, on p. 313, that it is not necessary to suppose that adjustments occur over a «long» period of time: what is essential is only that there be continuity in important for what we have said in para. 7-8. Pareto too presents the process of iteration (in worth. For instance it is not clear that the moves of the first individual aiming at an improvegroup of contractors (buying or selling a certain good).

⁴⁴ It is assumed that the quantities of the same good offered by other sellers do not change and that the buyers in turn do not engage in operations of a monopolistic kind.

But this is just a theoretical conclusion which must be corrected and adapted to what happens in practise. In fact it does not occur (as it should instead happen in a world of perfect calculators) that single individuals even though few in number, have the same attitudes and *animus* as the monopolist. In the exchange of goods for money especially the buyers are likely to accept the constant market prices and do not attempt with subtle devices to force the sellers to reduce their profits. The above-mentioned proposition therefore must be seen as having limited validity and capable only of showing a tendency.

10. In particular, those forms of price discrimination which have their source in an artificial division of the consumer into several classes, do not represent exceptions to the practice of constant prices. In these instances it is true that goods or services which are not «essentially» different from one another are offered to the consumers with a considerable difference in price which is disproportionate to the presumed difference in cost. However, the different prices do not concern identical units of the same good sold to the same individual but units of apparently different goods; among which consumers have in general ample freedom of choice, in accordance with the desire for social distinction.

The different qualities of the good appear then to be substitute goods, adding however that the very discrimination in price may induce some to acquire the good of a higher quality. Moreover, individual demand is influenced by the fact that there may be sold all together on the market greater or lesser quantities of different kinds of the same good: so that it is difficult to set separate demand curves for each of them. At any rate if it is considered legitimate to construct individual demand curves, one should not expect the sum of such curves (added over the abscissa) to be equivalent to the single curve that would obtain had the same good been sold in only one kind of quality⁴⁵. So

⁴⁵ On this subject, a letter sent to me by Prof. Ulisse Gobbi presents some typical hypotheses which may usefully be compared to those of Pigou (*The Economics of Welfare*, cit., part II, ch. XVII: «Discriminating Monopoly». See especially sect. 5 and ff.): «How is the demand affected by the application of variable prices to groups of consumers rather than a price equal for all:

A) Products of which each consumer buys one unit regard less of the price until the price has reached a level beyond which he no longer buys.

If the price which is the same for all is OA (see Fig. 1), the demand is OB; if it is OC, the demand is OD; if it is OE, the demand is OF.

Let the consumers be divided into three groups, applying to them respectively prices OA, OC, OE.

The following distinction must be made: a) the consumers cannot choose the group in which they are placed (e.g. piano lessons at a different cost according to the wealth of the family, discounts granted to members of certain associations, to students of certain schools, servicemen, etc.); b) or they may be able to choose it (better seats in a movie theatre, luxury and average shops, etc.).

much for demand. As regards supply, let us leave aside the numerous problems on the conditions and forms of discrimination. Then it is sufficient to recall that a monopolistic power, on the part of the seller, is necessary whenever price discrimination is enacted by a non-profitable entity which merely provides increased advantage for the collectivity⁴⁶.

In hypothesis A a (except for the negligible possibility that one might become annoyed because he is not given the same discount given to others), the quantity sold (part at the price OA, part at the price OC and part at the price OE) is equal to that sold at the single price OA. In general, the quantity sold at variable prices is equal to that sold at the single price corresponding to the minimum of the variable prices.



In hypothesis A b) the quantity sold at variable prices is still equal to that sold at a single price corresponding to the minimum of the variable prices, but at the price OE there will be a sale of $EL_1 < EL$, that is the quantity sold at the higher price will be less than what would be sold at that price if it were the only price; the quantity sold at the minimum price will be greater than that of hypothesis A a); the quantity sold at the intermediate price increases on one side and decreases on the other although we do not know in what proportion.

B) Products for which an increase in price decreases the quantity purchased by single individual consumers: the quantity sold at variable prices is *less* than that sold at a single price corresponding to the minimum of the variable prices.

C) Products which some consumers buy only if they can distinguish themselves from groups of consumers they consider inferior (so that the price that is reasonable for the former cannot be such for the latter): the quantity sold at variable prices may be *higher* than that sold at a single price corresponding to the minimum of the variable prices».

If we admit – granted that it may be debatable – that for every quality of the good question there is a demand curve *independent of the others* the problem may be treated analogously to that of several markets *not communicating among themselves* in which a monopolist producer is selling a given good; for this, see the equations of equilibrium in Amoroso, *Lezioni di economia matematica*, p. 271, and in Pigou, *op. cit.*, footnote p. 302, where, however, the general formula is incorrect probably due to a printing error (the {}) should be omitted).

⁴⁶ To compare the utilities the authors generally employ Marshallian rents for both consumers and the producer; this method presents problems (cfr. sect. 5); but in any case the following conclusion by Pigou with which Edgeworth agrees (*Papers*, vol. II, p. 445) is worthy note: if the selling price of two goods (or different qualities of the same good) is the same and is equal to the marginal cost of both, the situation is preferable to any adjustment to unequal prices. In this regard see other statements by Edgeworth in vol. I of his *Papers*, p. 101. But beyond the price discrimination which are made by presenting the good in different forms, there may also be variation in the prices for successive identical *units purchased by the same individual*⁴⁷: for example, this may occur in the supply of electric power. It is nonetheless important to stress that this occurs in exceptional cases, and that the most important case is mainly of theoretical interest in that the application of variable prices ensures the socialist State the maximum of Pareto's ophelimity for the collectivity⁴⁸. It must also be said that all these cases are characterised by the *unilateral* imposition of price regime. So that if the equilibrium is stable it is confirmed that variable prices cannot follow from the free exercise of individual forces, and therefore if variable prices are used they must be considered (as a type of price) among the *givens* of the economic system.

11. Let's now give a formal treatment of individual demand. It is well known that the ophelimity index-function that has to be assumed does neither refer only to consumption goods, nor to a single typical span of time: as a matter of fact, both labour supply and the demand and supply of savings are in a direct relationship with tastes and individual sensitivity. Since it is possible to deal separately with these phenomena, we will leave them out of account. Labour supply is indeed bound to customs and legal norms and, besides, unexplicable in terms of hedonistic considerations until the worker obtains what is needed to survive. Furthermore, as far as savings are concerned, it is easy to apply the concept of individual equilibrium, recurring to an ophelimity function whose arguments are the current and future income quantities that are foreseen within the bounds of the individual *economic horizon*⁴⁹.

If we consider only a span of time and assume that monetary income is known, we will study firstly its sharing among the purchasing of a certain quantity of good A_1 and the purchase of the other goods A_2 , A_3 , ... A_n that are all included in residual monetary income: we will reconnect afterwards to the more general treatment of consumer's equilibrium. This procedure is often sketched in the literature⁵⁰, but its details are not adequately developed.

⁴⁷ P. Jannaccone, in the quoted article: Il «dumping» e la discriminazione dei prezzi (pp. 256 and ff.), sharply distinguishes variable prices in the sense just mentioned from other forms of discrimination of prices.

⁴⁸ Pareto, Manuale di economia politica, cit., VI, pp. 58-59.

⁴⁹ The notion was introduced by J. Tinbergen, The notion of horizon and expectancy in dynamic economics, *Econometrica*, July, 1933. We can introduce this notion into a static framework unless it is intended to link successive equilibria. Analogously, Frisch (*op. cit.*, pp. 136 and ff.) does not abandon the static framework when he assumes (that the utility of income is a function of its *actual* amount and of the income own on the average in a certain previous period (the last five or ten years).

⁵⁰ See Pareto, Manuale, cit., IV, 64; Borgatta, Di una proprietà generale dell'ofelimità, cit., p. 31.



The equilibrium condition is given by Jevon's theorem (under the particular conditions that will be later specified): in our case, we can assume that this position is reached and study the shifts from it. It is worthy to note that, even if we suppose that indifference curves are known, we cannot take into account how the increase or decrease of the sales of good A_1 (in the market) affect individual tastes⁵¹.

In Fig. 2 are represented: in the ordinate axes the monetary income own by the individual (OB) and in the abscisses the various quantities of good A_1 that are demanded at the prices represented by downward sloping segments with origin in *B*. The equilibrium price is given by the slope of *BC*: at *C*, the path becomes tangent to an indifference curve, and there the individual stops, purchasing the quantity *OE* at a total expenditure of *BD*. If now the price or the monetary income changes, the demanded quantity will change accordingly to the shape of the indifference curves in the neighborhood of *C*. Let's now focus the attention on the two different cases, supposing that variations are infinitesimal so that the analysis might be profitable.

⁵¹ To overcome this problem, Pigou distinguishes the «demand price» from the «marginal demand price»: this distinction however is not so sharp, since the «marginal demand price» is not the same accross consumers (whenever it differs from the first). See Edgeworth, *Papers*, vol. II, pp. 323 and ff. A) Price variation. Let's denote with F(x, y) the function mapping the indifference curves. Along each of these F(x, y) = constant and at each point of the plane, the slope of the indifference curve is $\frac{dy}{dx} = y' = -\frac{f_x}{f_y}$, where f_x , f_y , are the partial derivatives of F with respect to x and y. At a given point, denoting with f_{yy} , f_{xx} and $f_{yx} = f_{xy}$ the second derivatives, we get

[1]
$$\frac{\partial y'}{\partial y} = -\frac{y' f_{yy} + f_{xy}}{f_y}$$

[2]
$$\frac{\partial y'}{\partial x} = -\frac{f_{xx} + y' f_{xy}}{f_y}.$$

Since y' is a function of x and y, each variation dy' is related to the variations dx, dy through the relation

$$[3] dy' = \frac{\partial y'}{\partial x} dx + \frac{\partial y'}{\partial y} dy,$$

from which we can recover the condition to remain on the same indifference curve, that is:

[4]
$$\frac{\delta y'}{\delta x} = y'' = \frac{\partial y'}{\partial x} + y' \frac{\partial y'}{\partial y}.$$

That being stated⁵², let's point out that, at the equilibrium point C (where x = OE, y = OD), the price p_1 of good A_1 , that for the time being is simply denoted by p, is equal to -y'. That is why, given a price variation dp, we can consider instead -dy'. Denoting with p' the ratio $\frac{1}{\partial x}$ and reading [3] we get:

$$-p' = \frac{\partial y'}{\partial x} + \frac{\partial y'}{\partial y} \frac{d y}{d x}.$$

⁵² About this, see Pareto, *Manuale*, cit., App. §§ 6-11. We try to follow as much as possible his notations.

Total monetary income ρ is defined by $\rho = px + y$, and therefore, supposing ρ constant, $\frac{dy}{dx} = -p - p'x$. Substituting in the previous expression $\frac{dy}{dx}$ with its value we get

$$\frac{\partial y'}{\partial x} + \frac{\partial y'}{\partial y}y' = p'\left(\frac{\partial y'}{\partial y}x - 1\right).$$

Consequently, the ratio between the variation of the demanded quantity and the supposed price variation is³³:

$$[5] \qquad \qquad \frac{\partial x}{\partial p} = \frac{\frac{\partial y'}{\partial y} x - 1}{y''}.$$

B) Income variation. In this case, y' does not change, but the line BC shifts parallely. We can write:

$$\frac{\partial y'}{\partial x}dx + \frac{\partial y'}{\partial y}dy = 0,$$

where the relation $pdx + dy = d\rho$ is verified, and therefore $\frac{1}{\frac{\partial x}{\partial \rho}} - p = \frac{d y}{d x}$.

Substituting it in the previous expression and reading [4], we get:

$$\frac{\frac{1}{\partial x}}{\frac{\partial \rho}{\partial \rho}} = -\frac{y''}{\frac{\partial y'}{\partial y}},$$

⁵³ Analogous expressions are found in W.E. Johnson, The pure theory of utility curves, *Economic Journal*, December, 1913, for the share of a given amount of money in the purchase of *two* goods: in particular see p. 504.

$$[6] \qquad \qquad \frac{\partial x}{\partial \rho} = -\frac{\frac{\partial y}{\partial y}}{y''}$$

If we compare these expression with [5], we see that $\frac{\partial x}{\partial p}$ and $\frac{\partial x}{\partial \rho}$ are linked by the relation

[7]
$$\frac{\partial x}{\partial p} = -\frac{1}{y''} - x \frac{\partial x}{\partial p}.$$

12. The expressions we obtained are relatively simpler than those presented by other authors⁵⁴. They get rid of any consideration about utility as measurable quantity and can easily adapt to the various hypothesis about independency of the utilities of single goods.

Expression [7] shows that in general $\frac{\partial x}{\partial p}$ and $\frac{\partial x}{\partial \rho}$ assume different values (and, as it will be shown in the following, it may sometimes not be of opposite sign). Furthermore, Frisch proved³⁵, contrary to the opinion of Marschak, that is impossible that even *approximately*, excluding particular instances, the two elasticities $\frac{\partial x}{\partial p} \cdot \frac{p}{x}$ and $-\frac{\partial x}{\partial \rho} \cdot \frac{\rho}{x}$ be equal. Taking for granted Fisch's conclusion, even if it is grounded on the hypothesis of independent utilities, we now briefly sketch another procedure to express the relation between $\frac{\partial x}{\partial p}$ and $\frac{\partial x}{\partial \rho}$. Obviously – tastes remaining the same – a given variation of income is followed by a variation in consumption (of good

⁵⁴ As far as $\frac{\partial x}{\partial p}$ is concerned, see Pareto, Considerazioni sui principi fondamentali dell'economia politica pura, *Giornale degli economisti*, 1892-93, in particular October 1893, pp. 304-306 and *Manuel*, cit., App. §§ 52-53. See also, about the hypothesis of indipendent goods, E. Barone, A proposito delle indagini di Fischer, *Giornale degli Economisti*, May 1894, p. 434; Ricci, Curve crescenti di ofelimità elementare e di domanda, *Giornale degli Economisti*, August 1904, p. 133 and The psycological foundation of the law of demand, *Journal of Political Economy*, 1932, pp. 180 and ff.

⁵⁵ Revue d'Economie Politique, 1932, pp. 14-28. If the good considered absorbs a very small fraction of income, the more the two elasticities differ the more the absolute value of the flexibility of marginal utility of income is far from one.

 A_1 , for instance) equal to the one that would occur in the case of a corresponding *proportional* variation, in the opposite direction, in the prices for all goods: we call p_1 and x_1 respectively the price and the demanded quantity of good A_1 (up to now denoted by p and x) and p_2 , p_3 , ... p_n the prices for goods A_2 , A_3 , ... A_n ; if

$$\frac{\partial \rho}{\rho} = \frac{d p_1}{p_1} = \frac{d p_2}{p_2} = \dots = \frac{d p_n}{p_n},$$

we yield:

$$-\frac{\partial x_1}{\partial \rho}d \rho = \frac{\partial x_1}{\partial p_1}dp_1 + \frac{\partial x_1}{\partial p_2}dp_2 + \dots + \frac{\partial x_1}{\partial p_n}dp_n,$$

Given the previous inequalities:

$$-\frac{\partial x_1}{\partial \rho} = \frac{\partial x_1}{\partial p_1} \frac{p_1}{\rho} + \frac{\partial x_1}{\partial p_2} \frac{p_2}{\rho} + \dots + \frac{\partial x_1}{\partial p_n} \frac{p_n}{\rho}.$$

In a simpler form, if instead of p_2 , p_3 , ... p_n we consider a price index P, we obtain:

[8]
$$-\frac{\partial x_1}{\partial \rho} = \frac{\partial x_1}{\partial p_1} \frac{p_1}{\rho} + \frac{\partial x_1}{\partial P} \frac{P}{\rho}.$$

Studying this expression we can conclude the following:

1) If p_1 and P vary proportionally, the ratio between the variation of xand the variation of p_1 is given by $-\frac{\partial x_1}{\partial \rho} = \frac{\rho}{p_1}$. If furthermore we take into account a variation of ρ , such that $\frac{d \rho}{k} = \frac{d p_1}{p_1}$ then $\frac{d x_1}{d p_1} = \frac{\partial x_1}{\partial \rho} \frac{k - \rho}{p_1}$. 2) If variations in p_1 , P, and ρ such that $\frac{d p_1}{k} = \frac{d P}{P} = \frac{d \rho}{\rho}$ $(k \neq p_1)$ occur, it is easy to verify that the ratio $\frac{dx_1}{dp_1}$ is equal to $\frac{\partial x_1}{\partial p_1} \left(1 - \frac{p_1}{k}\right)$. If instead $\frac{dP}{k} = \frac{dp_1}{p_1} = \frac{d\rho}{\rho}$, $(k \neq P)$, we get $\frac{dx_1}{dp_1} = \frac{\partial x_1}{\partial P} \frac{k-P}{p}$.

3) In any other case, in order to determine empirically the value $\frac{\partial x_1}{\partial p_1}$,

 $\frac{\partial x_1}{\partial P}$, $\frac{\partial x_1}{\partial \rho}$, it is necessary to know the two groups of variations (with respect to the initial situation) of p_1 , P and ρ , along with the corresponding variations of x. We can then establish and solve, making use also of equation [8], a system of three non homogenous linear equations with three unknowns.

13. The conclusions drawn above are important to carry out empirical investigations. As far economic theory is concerned, it is more useful to focus the attention on [5] and [6], trying to reconnect them to the more articulated formulas in which all goods are explicitly taken into account.

A general condition we have to impose for the position from which we started (point C) to be a stable equilibrium concerns the sign of y''. If this sign were negative, that is if the indifference curve at C were concave with respect to the axes, the consumer, overtaking C along the linear exchange path, would succeed in improving her ophelimity index: since we exclude that the consumer moves from C if no perturbation occurs, we have to assume that y'' is positive.

This happens for sure⁵⁶ when the two following disequalities are verified:

[9]
$$f_{xx} < 0, \quad f_{xx} f_{yy} - f_{xy}^2 > 0,$$

the second one imples that, since $f_{xx} < 0$, also $f_{yy} < 0$. As a matter of fact, y" can take the following form:

$$y'' = -\frac{1}{f_y f_{yy}} [(y' f_{yy} + f_{xy})^2 + f_{xx} f_{yy} - f_{xy}^2].$$

We immediatly verify that the positive value for y'' is yielded from the previous disequalities. They characterize what therefore can be called the *normal* case.

56 Pareto, Manuel, cit., p. 577.

This includes two subcases:

1) $\frac{\partial y'}{\partial x} > 0$, $\frac{\partial y'}{\partial x} < 0$ (see equations [1] and [2]); in this case it is obvi-

ous for [4], since y' < 0, that y'' > 0

2) one of the two disequalities written above is not satisfied. We can exclude a priori that none of them is verified, since this would imply $y'' < 0^{57}$.

Let's start supposing that $\frac{\partial y'}{\partial y} > 0$. By [6], then, $\frac{\partial x}{\partial y} < 0$. Since $\frac{\partial y'}{\partial y} > 0$, if $\frac{\partial y'}{\partial y} x - 1 > 0$, the expression $\frac{\partial x}{\partial p}$ turns positive⁵⁸. On the contrary, every time that $\frac{\partial y'}{\partial y}$ is negative, we get $\frac{\partial x}{\partial x} > 0$,

 $\frac{\partial x}{\partial p} < 0$. As long as $\frac{\partial y'}{\partial x} > \frac{p}{x}$, demand elasticity is, in absolute value, less then one: as a matter of fact (observing [4]), $-\frac{\partial x}{\partial p}\frac{p}{x} = \frac{\frac{p}{x} - p\frac{\partial y'}{\partial y}}{y''}$ is \leq ac-

cording to whether $\frac{p}{r} + y' \frac{\partial y'}{\partial y} \leq \frac{\partial y'}{\partial r} + y' \frac{\partial y'}{\partial y}$ where we can suppress the

common quantity both at the left and the right hand. When $\frac{\partial y'}{\partial y} < 0$, the increase in consumption due to an increase in income is such to decrease the residual income (that is to absorb more than the increase in income): in fact,

if
$$-p\frac{\partial y'}{\partial y} > y''$$
, that is $\frac{\partial y'}{\partial x} < 0$, then $p\frac{\partial x}{\partial \rho} > 1$.

The relationships expressed for subcase 2) are sufficient to answer the

⁵⁷ Since y" > 0, the term $-\frac{1}{y''}$ in [7] is negative. This term expresses what E. Slutsky, in

a work outstandingly valuable, but difficult to understand, called «the residual variability of a good in the case of a compensated variation of its price» (Sulla teoria del bilancio del consumatore, Giornale degli Economisti, July 1915). If the increase in price is dp and income in-

creases of the quantity
$$xdp$$
, $dx = \frac{\partial x}{\partial p} d p + \frac{\partial x}{\partial \rho} x d p$, and therefore $\frac{d x}{d p} = -\frac{1}{y''}$.

About this relationship and the following ones, see Johnson, op. cit., pp. 505 and ff. 58

question whether an increase in price, *ceteris paribus*, could generate an increase in consumption. For this to come, it is necessary that $\frac{\partial y'}{\partial y}x - 1 > 0$.

Notice that, if $f_{xx} < 0$, $f_{yy} < 0$, $\frac{\partial y'}{\partial y}$ cannot be positive but when $f_{xy} < 0$ according to [1], and

$$[10] -f_{xy} > y'f_{yy}.$$

We exclude therefore that the mentioned eventuality occurs in the normal case, if the utilities of goods are supposed independent $(f_{xy} = 0)^{59}$: in this case, however, if the good considered and the residual income are substitutes, it may occur that $\frac{\partial x}{\partial p} > 0$. The elements that are responsible for this

occurence (in addition to the price level) are:

a) the so-called substitutability ratio, that decreases the marginal utility of the good (f_x) when the available quantity of money increases;

b) the (absolute) value of f_{yy} , that has not to be very high;

c) the absolute value of f_{xx} , that has to be relatively high, since $\frac{\partial y}{\partial x}$

must be positive.

It is likely that these conditions are present in the example that is frequently brought about in the literature⁶⁰, that is the consumption of bread for lower income classes. If we are intended to give account of this phenomenon within the framework of economic theory (divisibility of goods, absence of perturbing shocks, absence of speculation, etc.), we have to abandon the hypothesis of independence of utilities. While for higher level of income f_{xy} is positive for almost all goods and, if sometimes it is negative, it never exhibits a high absolute value, for lower level of income and when necessities, like bread, are considered, then f_{xy} is negative; on the other hand, bread, that is sold at low price, is generally consumed up to largely satisfy the corresponding need, so that f_{xx} (negative) is different from zero; f_{yy} , instead, reckoning the variations of utility of other goods purchased in small quantities, is close

to zero. With successive increases in prices it is anyway likely that $\frac{\partial x}{\partial p}$ turns to be again negative, since f_{xy} (in absolute value) decreases as y becomes neg-

⁵⁹ For the other cases, see § 18. Let's notice now that [10] implies $f_{yy} > 0$ if $f_{xy} = 0$.

⁶⁰ See, for instance, Marshall, Principles, cit., p. 132.

ligible; this is due to the fact that goods that could satisfy the need of nourishment better than bread (like meat and similar) do not enter anymore in the budget of the consumer, and this could eventually affect the value of f_{yy} lowering it, that is bringing it further from zero.

These esplanations, upon which many authors agree⁶¹, coadiuvate in completing the one recently provided by Ricci⁶² that was based upon imperfect divisibility of goods. As far as other formulations of the problem are concerned, especially with regard not to necessities, but to luxury goods⁶³ it seem difficult to study them in the general analytical framework, since it is admitted that variations in the price level and in the quantity sold in the market affect tastes, inducing for instance some individuals to reduce or annihilate completely consumption after a reduction in prices.

Apart from the normal case, it may happen - all the more reason - that

 $\frac{\partial y'}{\partial y} > 0$ (for which [10] must hold) or that $\frac{\partial y'}{\partial x} < 0$, with the consequenc-

es mentioned above. We will come back briefly later (§ 17) to the limits of non conventional hypothesis.

14. It is convenient to state precisely the meaning of f_{yy} , f_{xx} , f_{xy} . Obviously, unless we assume that utilities are independent, the sharing of y among wother goods» depends on the quantity of the good that we explicitly take into account. We have, therefore, to analyse the responses of the sharing of y due to a change in x and the effects of such responses (due possibly to a simple increase in y) on marginal utility f_x . If we move in this direction, we reconduct to the general case introduced by Pareto: it is therefore useful to prove first of all that F(x,y) is just an aspect of the ophelimity index function $\Phi(x_1, x_2, ..., x_n)$. In F, x represents the quantity of a given good; in §§ 11 and 12 we defined A_1 ; we now introduce the notation A_{ij} (i = 1, 2, ..., n), and x becomes x_{ij} . We write, moreover, $y''_{(i)}$, f_{x_i,x_j} or more simply, f_{ii} , $f_{yy(i)}$, f_{x_iy} to indicate that we consider the good A_{ij} . It is now clear that in F, that take as given the prices p_1 , p_2 , p_{i-1} , p_{i+1} , ... p_n , the combination of the quantity x_i

⁶¹ Pareto, Cours d'économie politique, Lausanne, 1896, vol. II, § 977, and Manuel, cit., App. §§ 55; Johnson, op. cit., p. 506 (where a reduction of the price is envisaged); Slutsky, op.

cit., p. 14. It for a given good $\frac{\partial x}{\partial \rho}$ is negative, Slutsky defines that good «relatively dispensable

(or not indispensable)», while Johnson claims that the needs y satisfies are relatively «urgent».

62 See Può una curva di domanda esser crescente?, Giornale degli Economisti, 1932, p. 240.

63 Amoroso, Lezione di economia matematica, p. 151; see also Cournot, Principii matematici della teoria delle ricchezze, Biblioteca dell'economista, Serie III, vol. III, p. 96. (variable) and of the quantities $x_1, x_2, x_{i-1}, x_{i+1}, \dots, x_n$ that are feasible at those prices and given the amount of money y (variable) are ranked in the same preference ordering as in Φ . We can therefore write, for each of these combinations $f_{x_i} = \varphi_{x_i}$ (or also φ_i), $f_y = \varphi_y$, indicating with φ_y the weighted elementary ophelimity of the goods $A_1, A_2, A_{i-1}, A_{i+1}, \dots, A_n$: at equilibrium $\varphi_y = m$ (elementary ophelimity of income) holds.

Let's consider firstly the effect of an increase of y on the equilibrium position. We notice that this increase cannot occur if x_i remains constant; it is anyhow necessary to introduce this hypothesis in order to derive the value of $f_{yy(i)}$ (that can be written, following the previous notation, in the form $\varphi_{yy(i)}$). For sake of clarity, suppose that the increase of y takes the form of a gift, that takes place only if the subsequent purchase of A_i does not differ from the previous one. We have n-1 different, but competing ways to define $\varphi_{yy(i)}$, according to whether we consider the change in weighted elementary ophelimity of $A_1, A_2, A_{i-1}, A_{i+1}, \dots A_n$: since $d\varphi_j = p_j d\varphi_j$, must hold always $(j = 1, 2, \dots i - 1, i + 1, \dots n)$ we can write the following system of line-

ar equations in which $\frac{\partial x_j}{\partial y}$ are the unknowns:

$$\varphi_{11}\frac{\partial x_1}{\partial y} + \varphi_{12}\frac{\partial x_2}{\partial y} + \dots + \varphi_{1, t-1}\frac{\partial x_{t-1}}{\partial y} + \varphi_{1, t+1}\frac{\partial x_{t+1}}{\partial y} + \dots + \varphi_{1n}\frac{\partial x_n}{\partial y} = p_1 \varphi_{yy(t)}$$

$$\varphi_{21}\frac{\partial x_1}{\partial y} + \varphi_{22}\frac{\partial x_2}{\partial y} + \dots + \varphi_{2,i-1}\frac{\partial x_{i-1}}{\partial y} + \varphi_{2,i+1}\frac{\partial x_{i+1}}{\partial y} + \dots + \varphi_{2n}\frac{\partial x_n}{\partial y} = p_2 \varphi_{yy(i)}$$

$$\varphi_{n1}\frac{\partial x_1}{\partial y} + \varphi_{n2}\frac{\partial x_2}{\partial y} + \dots + \varphi_{n,i-1}\frac{\partial x_{i-1}}{\partial y} + \varphi_{n,i+1}\frac{\partial x_{i+1}}{\partial y} + \dots + \varphi_{nn}\frac{\partial x_n}{\partial y} = p_n \varphi_{\mathcal{D}(i)}$$

In these equations, we indicate with $\varphi_{11}, \varphi_{12}, ...$ the second derivatives of Φ with respect to the goods whose index is the subscript of φ : as we know, $\varphi_{12} = \varphi_{21}, \varphi_{13} = \varphi_{31}, ...$

To solve the system, we calculate the determinant that is made of the coefficients. But since in the following we shall refer to other determinants that

are *minors*, as this one, of the same determinant, we write this last One^{64} : we call M the determinant

in which the numbering of rows and columns starts from zero, in order to keep the correspondence between the index of the rows and columns with the index of the goods they consider. It is a symmetrical determinant from which, suppressing the 0-index row and column and the negative sign, we derive the functional determinant R made of all second derivatives of Φ . We will denote a minor of M (or of R), obtained by suppressing some rows or columns, by the same letter M (or R) subscribing the index of rows and columns suppressed, one beneath the other if two o more rows and columns are suppressed; in any case, the sign of M (or R) changes when the sum of such indexes is an odd number. With these warnings, it becomes clear that $R = -M_{00}$.

At a first stage, it is worthy to notice that M_{0j} is equal to the determinant that is obtained substituting in the *j*-th column of R the prices $p_1, p_2, \dots p_n$ instead of $\varphi_{j1}, \varphi_{j2}, \dots \varphi_{jn}$; if a similar substitution is carried out in the determinant R_{ii} , we get $-M_{0i}$. By Cramer rule, we obtain for the system above:

⁶⁴ In spite of the similarities with the treatment by Pareto (*Manuel*, cit., App. § 52), our procedure has distinctive features, according to the principal aim of the theoretical reconstruction that is herein sketched: 1) show the link between the simpler solution of the problem of equilibrium (for a given good in relation to residual income) and the more articulated and general solution; 2) define stability condition in a more direct way compared to the one proposed by Slutsky (*op. cit.*): in our formulation only the knowledge of the basic principles on determinants (minor formation and expansion methods) and of the Cramer Rule for the solution of a system of linear equations is required.

In a very recent essay by H. Shultz (Interrelations of demand, Journal of Political Economy, August 1933), the problem of the equilibrium of the consumer is treated dealing only partially with the features of the ophelimity index function (see specially pp. 477-481). These features are at a first stage brought into the analysis, but later it is assumed that marginal utility of income remains constant; furthermore linear demand functions (or at least of a predetermined type) are assumed; this hypothesis cannot be easily justified from a theoretical point of view, but is corroborated by the empirical investigations that are carried out by the author.

[11]
$$\frac{\partial x_j}{\partial y} = -\frac{M_{oi}}{\frac{ij}{R_{ii}}}\varphi_{yy(i)}$$

Since
$$y = \sum_{j} p_{j} x_{j}$$
, that is $\sum_{j} p_{j} \frac{x_{j}}{y} = 1$, we yield:

$$\sum_{j} p_{j} \frac{\partial x_{j}}{\partial y} = 1,$$

and reading [11],

$$-\frac{\varphi_{yy(i)}}{R_{ii}}\sum_{j}p_{j} M_{oi} = 1,$$

[12]
$$w_{yy(i)} = -\frac{R_{ii}}{\sum_{j} p_{j} M_{oi}} = \frac{R_{ii}}{M_{ii}}.$$

15. We can define $f_{x_iy}(\text{or }\varphi_{x_iy})$ as the ratio between the change in φ_x , due to an infinitesimal increase in y, and the increase itself, or as the ratio between the change in φ_y , due to an infinitesimal increase in x, and the increase itself. The two definitions coincide for the general principle that imposes the equality of the two second cross-derivatives (continous at a given point) of a two-variable function; we will use the first, that we find more tractable, to find the value of φ_{x_iy} . It is anyway possible to show this identity calculating φ_{yx_i} if we set up the problem according to the way indicated at the beginning of the next section (in which however we still write φ_{x_iy}).

We yield:

$$\varphi_{x_i y} = \varphi_{i1} \frac{\partial x_1}{\partial y} + \varphi_{i2} \frac{\partial x_2}{\partial y} + \dots$$

$$\dots + \varphi_{i, i-1} \frac{\partial x_{i-1}}{\partial y} + \varphi_{i, i+1} \frac{\partial x_{i+1}}{\partial y} + \dots + \varphi_{i, n} \frac{\partial x_{n}}{\partial y}$$

where, for [11] and [12],

$$\frac{\partial x_{j}}{\partial y} = -\frac{M_{oi}}{M_{ii}} \quad (j = 1, 2, ..., i - 1, i + 1 ..., n).$$

It follows

$$\varphi_{x_i,y} = -\frac{1}{M_{ii}} \sum_{j} \varphi_{ij} M_{oi};$$

and remarking that

[13]
$$M_{oi} = -p_i M_{oi} + \sum_j \varphi_{ij} M_{oi},$$

we obtain eventually

[14]
$$\varphi_{x_iy} = -\frac{M_{oi} + p_i M_{oi}}{M_{ii}} = \frac{p_i R_{ii} - M_{oi}}{M_{ii}}.$$

16. It remains to find f_{ii} , which is different from φ_{ii} since an increase in x_i determines changes in the sharing of y among all remaining goods; we have therefore to find the changes in $x_1, x_2, \dots, x_{i-1}, x_{i+1}, x_n$. Here, as in § 14, we can write n-1 equations to define φ_{yx_i} (that is equal to φ_{x_iy}); the first of them is:

$$\varphi_{11}\frac{\partial x_1}{\partial x_i} + \varphi_{12}\frac{\partial x_2}{\partial x_i} + \dots + \varphi_{1, i-1}\frac{\partial x_{i-1}}{\partial x_i} + \varphi_{1, i+1}\frac{\partial x_{i+1}}{\partial x_i} + \dots$$
$$\dots + \varphi_{1n}\frac{\partial x_n}{\partial x_i} = p_1 \varphi_{x_i y} - \varphi_{1i};$$

and it is easy to write analogous equations for the other goods. By Cramer rule, we obtain:

[15]
$$\frac{\partial x_j}{\partial x_i} = \frac{1}{R_{ii}} (-M_{oi} \varphi_{x_i y} - H_{(j)}),$$

where $H_{(j)}$ is the determinant obtained substituting in R_{ii} each element of the *j*-th column (according to the numbering of rows and columns in the determinant R) with the element that belongs to the same row and column *i*; we easily see that $H_{(j)} = M_{oj}$. After having multiplied both sides in [15] for φ_{ij} , we take the sum and obtain:

$$\sum_{j} \varphi_{ij} \frac{\partial x_{j}}{\partial x_{i}} = -\frac{\sum_{j} \varphi_{ij} M_{oj}}{R_{ii}} \varphi_{x,y} - \frac{\sum_{j} \varphi_{ij} M_{oj}}{R_{ij}}$$

We can now notice that $-\sum_{j} \varphi_{ij} M_{oj} = \sum_{j} \varphi_{ij} R_{ij} = R - \varphi_{ij} R_{ii}$;, reading [13] and [14], we yield:

$$\sum_{j} \varphi_{ij} \frac{\partial x_{j}}{\partial x_{i}} = \varphi_{x_{i}y} \frac{M_{ii}}{R_{ii}} \varphi_{x_{i}y} + \frac{R}{R_{ii}} - \varphi_{ii}$$

and consequently

$$f_{ii} = \overline{\varphi}_{ii} = \varphi_{ii} + \sum_{j} \varphi_{ij} \frac{\partial x_{j}}{\partial x_{i}} = \varphi_{x_{i}}^{2}, \frac{M_{ii}}{R_{ii}} + \frac{R}{R_{ii}}.$$

The final expression can take the following form:

$$\overline{\varphi}_{ii} = \left(p_i + \frac{M_{oi}}{M_{oi}}_{oi}\right)\varphi_{x_iy} + \frac{M_{oo}}{M_{oi}}_{oi} = p_i\varphi_{x_iy} - \frac{p_i M_{oi}}{M_{ii}} - \frac{M_{oi}^2}{M_{ii} M_{oi}} + \frac{M_{oo}}{M_{oi}}_{oi}$$

Exploiting a property of adjunt determinants65,

⁶⁵ If we define M' the adjunt of the determinant M (that is the determinant whose elements are the algebraic complements of the omologous of M) the following proposition holds (we quote from L. Berzolari, Determinanti, *Enciclopedia delle matematiche elementari*, Milano, Hoepli, 1932, p. 87, adapting the quotation to our notation): «If Δ and Δ' are two minor of order b omologous in M and M' (that means built up with horizontals and verticals occupying the same places in M and M'), Δ' is equal to the product of M^{b-1} with the algebraic complements of Δ ». From this proposition, if b = 2 and rows and columns of M to which the elements of Δ belong have – the ones and the others – the index 0, i then [15] follows immediatly. (It is important to keep in mind that M is a symmetrical determinant, so that $M_{oi} = M_{io}$). Slutsky, in the above mentioned work (§§ 5 and 7), make skilfully use of this theorem.

[16]
$$M_{oo} M_{ii} - M_{oi}^2 = M M_{oi},$$

we get

[17]
$$\overline{\varphi}_{ii} = p_i \, \varphi_{x_i y} - \frac{p_i \, M_{oi}}{M_{ii}} + \frac{M}{M_{ii}}.$$

17. Reading the values in sections 14-16, the expression [1], [2], [3] become respectively (for good A_i)

$$\frac{1}{\varphi_{y}}\frac{M_{oi}}{M_{ii}}, -\frac{1}{\varphi_{y}}\left(y_{(i)}^{\prime}\frac{M_{oi}}{M_{ii}}+\frac{M}{M_{ii}}\right), -\frac{1}{\varphi_{y}}\frac{M}{M_{ii}},$$

where φ_{y} , since we are at equilibrium⁶⁶, is equal to *m* (marginal ophelimity of income) and $y'_{(i)} = -p_i$. If we plug the value found for the expressions [1] and [4], expressions [5] and [6], can be transformed in a way such to deal explicitly with all goods:

[18]
$$\frac{\partial x_i}{\partial p_i} = m \frac{M_{ii}}{M} - x_i \frac{M_{oi}}{M}; \qquad \frac{\partial x_i}{\partial \rho} = \frac{M_{oi}}{M}.$$

After such a transformation, the problem of the stability of equilibrium still emerges. Since y" must be positive, for each of the *n* goods⁶⁷, the stability condition becomes the following: $-\frac{1}{m}\frac{M}{M_{ii}} > 0$, that is, since *m* is positive by its own nature:

.

[19]
$$\frac{M}{M_{ii}} < 0 \quad (i = 1, 2, ..., n).$$

⁶⁶ For positions different from that of equilibrium, the arguments proposed still hold: it is just needed to substitute (in the determinant M also) the value $-y'_{(i)}$ with p_i .

⁶⁷ If for each of the *n* goods the indifference curves (Fig. 2) are at each point convex with respect to the axes, only the equilibrium position is unique. Amoroso, in the work Discussione del sistema di equazioni che definiscono l'equilibrio del consumatore, Annali d'economia, 1928, makes explicit use of this hypothesis.

When does this condition hold?

1) Let's consider this typical case. Let's define $R_{(i)}$ the minor of R that is obtained by suppressing the last n-1 rows and columns: the above condition holds if the sequence

[20] 1,
$$R_{(1)}, R_{(2)}, \dots R_{(n-1)}, R$$

exihibits only variations of sign $(R_{(1)} < 0, R_{(2)} > 0, ...)$. In fact, in this case:

a) «all principal minors of R are different from zero: positive or negative according to whether their order is even or odd»⁶⁸: in particular, therefore, R_{ii} and R are of opposit sign.

b) $\frac{M}{R}$ is negative. To give a proof of this statement, let's notice that $M_{(a)}$

the minor of M obtained analogously to R_{ω} if we apply the relationship [16] to M_{ω} instead of M, is in general given by:

$$M_{(i)} = \frac{M_{(i-1)} R_{(i)} + M_{o, i+1, \dots, n}^2}{R_{(i-1)}}$$

that is

[21]
$$\frac{M_{(i)}}{R_{(i)}} = \frac{M_{(i-1)}}{R_{(i-1)}} + \frac{M_{o,i+1,\dots,n}}{R_{(i-1)}}$$

This amounts to say⁶⁹ that $\frac{M_{(i)}}{R_{(i)}}$ can in any case be represented by a sum

of fractions which have positive numerator and denominator equal to the product of two subsequent minors in sequence [20]. If in this expressions only variations of sign are present, all denominators of the above fractions are negative; then $\frac{M_{(i)}}{R_{(i)}} < 0$, $\frac{M}{R} < 0$. In particular, since the number of variations or permanences of sign in sequence [20] is independent from the ranking of the good⁷⁰, each good A_i can be considered as last, so that $\frac{M_{ii}}{R_{ii}} < 0$.

68 Berzolari, op. cit., p. 150.

- 69 See Slutsky, op. cit., p. 9.
- ⁷⁰ Berzolari, op. cit., p. 148.

The second second second

It is straightforward to see that the present case coincides to the one that has been defined *normal* in § 13. In fact, the second expression in [9] corresponds to the following disequality:

$$\frac{R}{M_{ii}} > 0.$$

Being $\frac{R_{ii}}{M_{ii}} = \varphi_{yy_{(i)}}$ negative, the first in [9] is necessarily verified.

2) Apart from the normal case, if the elements of M assume particular values, it may still happen that disequality [19] holds for all goods, but this is very unlikely to come. It is worthy to mention just a single, exceptional case, which can be defined by simple generic propositions: it is the case where the sequence [20] exhibits just one permanence of sign, and furthermore $\frac{M}{p} > 0$. Suppose the permanence of sign to be between the minors

 $R_{(j-1)}$ and $R_{(j)}$. Then the fractions that form the addenda in the sum in which M can also are all possible but the one that presents as denominate the

 $\frac{M}{R}$ can change are all negative but the one that presents as denominator the

product $R_{(j-1)} R_j$. For b = 1, 2, ..., j-1, then $\frac{M_{(b)}}{R_{(b)}} < 0$; $\frac{M_{(j)}}{R_{(j)}}$ is instead posi-

tive since greater than $\frac{M}{R}$ that is positive: by the same token, the ratios

 $\frac{M_{(k)}}{R_{(k)}}$ where k = j + 1, ..., n - 1, are all positive. This is enough to claim that

the sequence $M_{(1)} = p_1^2$, $M_{(2)}$, ... $M_{(n-1)}$, M exhibits only variations of sign. Since this occurrence is independent from the ranking in which goods are considered, the disequality [19] still holds in this case. As far as the ratios

 $\frac{R}{R_{ii}}$ are concerned, it is not possible to state univocally if they are negative or positive in sign; it is certain (but we find it useless to prove) only that if

in any rank of goods $R_{(j-1)}$ $R_{(j)} > 0$, then for k > j, $\frac{R}{R_{kk}} < 0$. We can moreo-

ver notice that $\frac{R_{ii}}{M_{ii}} = \varphi_{\gamma\gamma(i)}$ has opposite sign to the one of $\frac{R}{R_{ii}}$, since 220

$$\frac{M}{M_{ii}} = \frac{R}{M_{ii}} \frac{M}{R}$$
 and therefore $\frac{R}{M_{ii}} < 0$ (opposite disequality to [22]).

18. When the utilities of single goods are supposed independent, the relationships mentioned above simplify considerably.

Let's denote w_x the flexibility of the marginal utility of the good, that is

 $\frac{x\varphi_{xx}}{\varphi_x} = -\frac{x}{p}\frac{\partial y'}{\partial x}$ and w_s the flexibility of the marginal utility (in most cases

increasing) of the money *spent*, that is $-\frac{x \ p \ \varphi_{yy}}{\varphi_{y}} = -x \frac{\partial \ y'}{\partial \ y}$. Expressions [4],

[5], and [6] become:

$$y'' = \frac{p}{x}(w_s - w_x)$$

$$\frac{\partial x}{\partial p} = \frac{x}{p} \frac{1 + w_s}{w_x - w_s}$$

$$\frac{\partial x}{\partial \rho} = -\frac{1}{p} \frac{w_s}{w_x - w_s}$$

Defining $\epsilon = \frac{\partial x}{\partial p} \frac{p}{x}$, the elasticity of demand, we get⁷¹.

$$\epsilon = \frac{1+w_s}{w_x - w_s}$$

We now discuss briefly this last expression.

⁷¹ R. Frisch (op. cit., p. 97) analysing labour supply, derives a similar expression, as W. Leontief, under particular assumptions, in Studien über die Elastizität des Angebots, Weltwirtschaftlisches Archiv, January 1932, p. 82.

Since y" must be greater than zero, that is $w_x - w_s < 0^{72}$ the sign of $\left(\frac{\partial x}{\partial p}\right)$ is opposite to the one of the numerator of the fraction above mentioned: similarly, the sign of $\frac{\partial x}{\partial \rho}$ is the same of w_s . If $w_s > 0$, that is $\varphi_{yy} < 0$, we get $\frac{\partial x}{\partial \rho} > 0$, $\varepsilon < 0$, even if w_x is positive (but smaller than w_s). If $w_s < 0$, that is $\varphi_{yy} > 0$, we get $\frac{\partial x}{\partial \rho} < 0$, and $\varepsilon \le 0$ according to whether $1 + w_s \le 0$.

As far as independent consumptions are concerned, in the normal case we have, for any good $w_x < 0$, and $w_s > 0$, apart from the normal case, when the sequence [20] exhibits only one permanence of sign (that is the case where the second derivatives φ_{ii} are all negative but one), we can consider: either the good with a positive second derivative of the utility, and then we have on one side $w_s > 0$, on the other (as it will become apparent from the forthcoming formula [26] $w_s > 0$, or any other good, and then these flexibilities will be opposite in sign (see again equation [26]). We therefore have to consider three hypotheses:

a) $w_x < 0$, $w_s > 0$. This is the normal case; taking into account the positive expression $-\frac{1}{w_x}$ (that represents the elasticity of the need, that is of the utility curve of the good at the equilibrium point), reading [23] it is easy to show that if $-\frac{1}{w_x} \ge 1$, than also

$$\begin{cases} -\epsilon \gtrless 1 \\ -\epsilon \lessgtr -\frac{1}{w_x}. \end{cases}$$

b) $w_x > 0$, $w_s > 0$, (and $w_s > w_x$). Then $-\varepsilon > 1$ holds: therefore, rising the price, the expenditure in the good considered decreases, and the consumption of the other goods increases.

c)
$$w_x < 0$$
, $w_s < 0$, (and $w_s > w_x$). $\frac{\partial x}{\partial p}$ might then be greater than zero; as

⁷² Ricci shows graphically this inequality in the paper Può una curva di domanda esser decrescente?, at section 41. a matter of fact, this occurrence does not give proper account of the real case of a variation of the price and of demanded quantity in the same direction.

19. Maintaining the hypothesis of independence of goods, and knowing the empirical values of $\frac{\partial x}{\partial \rho}$ and of ε for a given good (on this point, recall what has been stressed in § 12), it is easy to compute the values for w_x and w_y . Let's define λ the expression $-p \frac{\partial x}{\partial \rho} = \frac{w_y}{w_x - w_y}$. Reading [23], we get:

[24]
$$\begin{cases} w_x = \frac{1+\lambda}{\epsilon - \lambda} \\ w_s = \frac{\lambda}{\epsilon - \lambda} \end{cases}$$

At this stage, the elasticity of the marginal utility (*m*) of income at equilibrium can be determined; by [12], we yield $\varphi_{m(i)} = \frac{R_{ii}}{M_{ii}}$. If now, instead of considering the residual income spent in n-1 goods, we consider ρ spent in the purchase of all other *n* goods, we obtain analogously $\frac{\partial m}{\partial \rho} = \frac{R}{M}$. This is true in general: under the particular assumption of independent goods, we can write:

[25]
$$\frac{1}{\frac{\partial m}{\partial \rho}} = \frac{p_1^2}{\varphi_{11}} + \frac{p_2^2}{\varphi_{22}} + \dots + \frac{p_n^2}{\varphi_{nn}}$$

This stems directly from the following equalities

$$m = \frac{\varphi_1}{p_1} = \frac{\varphi_2}{p_2} = \dots = \frac{\varphi_n}{p_n}$$

deriving with respect to ρ and recalling that $d_{\rho} = p_1 dx_1 + p_2 dx_2 + ... + p_n dx_n$. If we now consider all goods but A_1 , it is worthy to notice that [25] yields the following equality:

[26]
$$\frac{1}{\varphi_{yy(i)}} = \sum_{j} \frac{p_{j}^{2}}{\varphi_{jj}} \quad (j = 1, 2, \dots, i - 1, i + 1, \dots, n)$$

from wich it can be inferred what has been above stated about the sign of w_i , for different goods, whenever one of them exhibits increasing marginal utility.

Plugging now the definitions for w_x and w_y (given at § 18 for the case of

a generic good) according to which $\frac{p^2}{\varphi_{xx}} = \frac{px}{mw_x}, \frac{1}{\varphi_{yy}} = -\frac{px}{mw_s}$, we obtain:

$$\frac{1}{\frac{\partial m}{\partial \rho}} = \frac{p^2}{\varphi_{xx}} + \frac{1}{\varphi_{yy}} = \frac{px}{m} \left(\frac{1}{w_x} - \frac{1}{w_s} \right)$$

and, once defined
$$w_{\rho}$$
 the flexibility $\frac{\partial m}{\partial \rho} \frac{\rho}{m}$

[27]
$$\frac{1}{w_{\rho}} = \frac{px}{\rho} \left(\frac{1}{w_{x}} - \frac{1}{w_{s}} \right).$$

If the hypothesis of independent utilities is well-grounded, the expression written above⁷³ together with [24] should lead in practice to approximately equal results, if it is applied for (average) different consumptions of an economically homogenous social rank.

20. Anyway, in order to determine w_{ρ} , it is not necessary to assume that

⁷³ The forthcoming expression [28] allows to obtain the ratio between $\frac{\partial m}{\partial \rho}$ and m in another way, when at least one good has utility independent from the available quantities of the other goods.

all goods have independent utilities; it suffices that just one good is such, for instance A_1 . In fact, if all second cross-derivates φ_{12} , φ_{13} ,..., φ_{1n} are zero, then $M_{oi}_{oi} = -\frac{M_{oi}}{p_i}$. Using [16], we derive the equality

$$\frac{R}{M} = \frac{M_{oi}}{M_{ii}} \left(\frac{1}{p_i} - \frac{M_{oi}}{M} \right).$$

that, given [18], omitting for sake of semplicity the index 1, becomes:

[28]
$$\frac{R}{M} = \frac{\partial m}{\partial \rho} = m \frac{\frac{\partial x}{\partial \rho} \left(\frac{1}{\rho} - \frac{\partial x}{\partial \rho}\right)}{\frac{\partial x}{\partial \rho} + x \frac{\partial x}{\partial \rho}}.$$

It is important to notice that, when the ratio between $\frac{R}{M}$ and *m* is known by [28], it becomes theoretically possible to determine the ratio between each second derivative of the ophelimity function and *m*: that function becomes therefore univocally known in the neighborhood of the equilibrium point (keeping arbitrary only the choice of the unit of measurement).

Given [18], the quantities
$$\frac{M_{oi}}{M}$$
, $m \frac{M_{ii}}{M}$ $(i = 1, 2, ..., n)$ are known or

knowable; and so are the quantities $m \frac{M_{ij}}{M}$ (j different from i), since

[29]
$$\frac{\partial x_i}{\partial p_i} = m \frac{M_{ij}}{M} - x_j \frac{M_{oi}}{M}.$$

Knowing these quantities, once a positive value is attributed to m (for instance, 1), we can obtain all the values – but one – of the determinant inverse to -M (each of them is given by the ratio between the algebraic complement of the omologous element in M and M itself). The only element that cannot be determined recurring to those quantities, the one belonging to

row 0 and column 0, $-\frac{R}{M}$ is given by [28]. Once all the elements of the determinant inverse to -M are known, we obtain all the elements of -M (or R) computing the inverse of such determinant.

Since it is not improbable to assume that at least one good, which may satisfy a primary need, exhibits an utility independent from other goods, then we can conclude that it is always possible to specify the ophelimity function (in a given area) by means of objective data (whenever available). is important to verify if those data might singularly be recovered by experience. We admit that this is not likely even in a long span of time. If we recur to the procedure that consists in an inquiry based upon hypothetical price and income variations, it is very likely to obtain inconsistent results, since the economic agent is able to meet real contingencies, but it is extremely uncertain that she can imagine separately, and consistently all possible variations. when just a few really occur (in a short span of time), and the most jointly the same time. In order to empirically determine the derivatives of marginal utilities, it is therefore necessary to follow approximate methodologies, like the ones just sketched in the previous section.

21. In conclusion, we turn again to the issue of substitutability and complementarity among goods. In this regard, we will carry out only some remarks concerning, firstly, the very definition of the ratio between these goods, secondly, the shifts from market equilibrium.

As far as the definition is concerned, a dilemma emerges: should we resort to subjective criteria, that are seldom observed in reality, or, on the contrary, to the outward behavior of the consumer? The question often arises, since the cases of *perfect* substitutability or complementerity are extremely rare; the distinction, apart from extreme cases, turns out to be inevitably fuzzy. A possible subjective criterion consists in reading the sign of the second cross-derivative of the ophelimity function with respect to the two goods considered; if this derivative is positive, the two goods are complementary; negative, they are substitutes. An objective criterion consists in studying the direction of price variations of two goods in the market, when the quantity supplied of one of the goods increases or decreases⁷⁴. With regard to the case of two goods in which all income is spent (or for a given good and residual income). With regard to the case of two goods in which all income spent (or for a given good and residual income), Johnson⁷⁵ proposed a methodology that was subjective, but that was based on objective grounds; ac-

⁷⁴ See H. Dalton, The inequality of incomes, London, 1925, p. 190.

⁷⁵ Ibidem, pp. 495 and ff.

cording to Johnson, the sign of the expression $\frac{\partial x}{\partial \rho}$ is positive if there is a

complementarity relationship between the goods, and is negative for one, but

positive for the other if the goods are substitutes. It seems that if one is intended to study the relationship between two goods, without leaving out of account the influence of all other goods, the best criterion is the one related to the sign of the so-called «residual variability of a good for a compensated variation in the price of the other good»⁷⁶, for the immediate evidence of its subjective meaning, and for its objective display. Reading [29], and recalling [18], if the price of the good A_i increases of the amount dp_i , and income increases of the quantity $x_i dp_i$ in order to fully compensate this price change,

the ratio between the change in consumption of A_j and the increase dp_j is

equal to $m\frac{M_{ij}}{M}$; now, when the sign of this ratio – the above-mentioned residual variability – is positive, we can claim to have discovered a substitutability relationship between goods (since simultaneously x_j decreases)⁷⁷.

As far as the analysis of the shifts from equilibrium is concerned, it is worthy to notice that if two or more goods are linked by a *particular* relationship of interdipendence, then it is not convenient to schedule a simple demand curve for each individual, but it is necessary to establish a more articulated relationship between the demanded quantities of each of the two goods (just to limit ourself to the simplest case) and the prices of both. Each individual presents therefore a couple of functions in two variables. This allows to determine the change in the market equilibrium, defining the total demanded quantities as sum of individually demanded quantities, expressing the coincidence of prices and marginal cost and indicating (in conformity to the hypothesis introduced) the total supplied quantities.

In this formulation, if the analytical framework is correctly specified, the factors affecting individual demand functions are not fully explained. We can understand from this why Fanno⁷⁸ has appropriately deepened the analysis of

⁷⁶ Slutsky, *op. cit.*, p. 15, where it is also stressed that these variations are reversible: one can reverse the two goods without changing the outcome.

⁷⁷ It is worthy to notice that in the special case in which all second derivatives of Φ obtained by deriving twice with respect to the same variable are negative, and the second cross-

derivatives are all null but φ_{ij} which is *negative*, the ratio $\frac{M_{ij}}{M}$ is certainly positive (when the

sequence [20] exhibits only variations of sign). If φ_{ij} is positive, the above-mentioned ratio might be positive or negative. It is certainly positive if φ_{ij} is null (in this case [20] exhibits necessarily only variations of sign).

⁷⁸ Contributo alla teoria dei beni succedanei, Annali di Economia, 1926.



these factors, assuming that links of interdipendence between the substitute goods considered exist (we pass over the above-mentioned difficulties inherent to the definition) and that the utilities of other goods are independent from them. The following graphical analysis captures the distintive features of the core of Fanno's theory:

Let's define X and Z two substitute goods, and with p_x and p_z their market prices. The individual reaches the equilibrium scheduling firstly a demand function for one of these goods, X for instance, under the hypothesis of inexistence of the other; according to the criterion shown in fig. 2 the demand curve (fig. 3) is BE (monetary income OB). For a given value of p_x , be 0A the demanded quantity. Let's plug (see the lower part of the figure)

system of indifference curves between X and Z: since 0'A' = 0A, let's draw

straight line from A' so that the ratio $\frac{DC}{CA'}$, indicating its slope on the 0'x

axes, be equal to $N = \frac{p_x}{p_t}$. Moving from A' along the line just drawn, the consumer does not alter the quantity y of residual income spent for the other goods; once we introduce the questionable hypothesis that it will become

profitable for the consumer to change the quantity of money y allotted for the other goods, it will be convenient for her to move up to D, where the line becomes tangent to an indifference curve. The purchased quantity of X is therefore 0'C, and the quantity of Z is CD. Fanno⁷⁹ schedules analogously comprehensive indifference curve for the market, but we are not sure that this procedure is absolutely correct⁸⁰. Leaving out of consideration this neglectable flaw, from the sketch just presented it becomes apparent that Fanno's reconstruction allows to grasp important theoretical findings, especially since it considers the influence of a higher or lower variability of the substitution ratio (equal to the slope of the indifference curve) in the neighborhood of the equilibrium point.

The long list of arguments that have been extensively developed or rapidly sketched may end with a final remark. Even if we limited ourselves to small shifts from equilibrium, we secceeded in giving account of the essence of the reckoning carried out by the consumer. Being so general, this reckoning is not restricted by *particular* psycological premises; the opinions of those who expect a revision of economic theory from the achievements of psycological disciplines are not well-grounded. However, the comparison among the different hedonistic situations is complex, in spite of the simplicity of the premises, both in the case of independence of consumption and in the more articulated case of interdependence. Since it is always necessary to refer to a static framework, we are not able and maybe we will never be able to ascertain this comparison in all details: we will be content with approximate confirmations, that will not add much to what personal introspection can suggest.

Nevertheless, the doctrines that hinge on utility constitute the philosophical branch of economics. They interpret, analyse, and systematise the economic concern of human behavior. As such, even though they may seem too obscure or subtles, they cannot be abandoned.

⁸⁰ We indicate here the reasons for our skepticism. Once we know and suppose fixed the starting points for all individuals, then we can legitimately deduct from a system of individual indifference curves a system of curves for the market; as a matter of fact, an indifference curve tangent to the line expressing the price corresponds to a slope of this price line, for each individual; such curves can be validly summed up according to the procedure indicated by Fanno. As far as substitute goods are concerned, the starting points (A' in fig. 3) change as P_x changes, and the indifference curve for the market cannot be drawn. It is however possible to maintain the graphical representation, that is undoubtedly useful, and schedule a system of lines each corresponding to a given value of the ratio N. We sketch here briefly this methodology. It is easy to draw the lines we have just mentioned for each individual: each of them intersects the indifference curves; in correspondance of each point the equilibrium may occur for a variable value of p_x (and consequently, since N is constant along the line, for a value of them – that is for each value of N – coordinates equal to the sum of individual coordinates correspond to each value of p_x .

⁷⁹ Op. cit., p. 385.